With a Grain of Salt: Investor Reactions to Uncertain News and (Non)Disclosure*

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Abstract: We examine how external news with uncertain precision influences investor beliefs, market prices, and corporate disclosures. We find that good external (and public) news is taken with a grain of salt—specifically, it is perceived as unlikely to be precise—confirming investor beliefs that nondisclosing managers are hiding unfavorable (private) information. As a result, better external news may paradoxically lead to lower market valuation. Overall, we find that, amid management silence, equilibrium stock prices are nonmonotonic in (and react asymmetrically to) external news. We also predict that the probability of disclosure depends on the timing of the disclosure, the positivity of external news, and the financial strength of the industry.

Keywords: uncertain precision, voluntary disclosure, price nonmonotonicity, asymmetric price reaction, probability of disclosure

JEL: D82, D83, G12

1 Introduction

When assessing a firm's value, investors increasingly rely on external public news, such as analyst forecasts, peer reports, and mainstream and social media, in addition to disclosures by the firm itself. However, the reliability of external news can be *uncertain*, as it is frequently unclear whether and to what extent a given news source is influenced by biases, rumors, errors, or even fraudulent intent. A notable case occurred on September 8, 2008, when Bloomberg News reported that United Airlines had filed for bankruptcy, and the market was uncertain whether this report was based on a court record, difficult-to-verify leaked documents, or imprecise rumors. Hours later, it became clear that the article had been generated in error (Maynard 2008; Schaper 2008; Carvalho, Klagge, and Moench 2009).[1] Incidents like this highlight the need for investors to take external news with a grain of salt. They also raise crucial questions: How does uncertainty about the precision of external public news influence a firm's decision to reveal its private information? And what are the implications of this uncertainty for market prices?

Our paper answers these questions in the context of a voluntary disclosure model. We study the implications of uncertainty regarding the precision of external news and show it can influence investor beliefs, corporate disclosures, and market prices in subtle ways. Several of our results diverge from prior work that does not account for belief updating about precision. Most notably, we find that investors are more skeptical of good news and that prices can be nonmonotonic (and asymmetrical). Our contribution is to explain how these phenomena occur as a direct consequence of the market's uncertainty about news' precision.

Our model features a manager (she), who may be endowed with private information about her firm's value and, if so, can remain silent or truthfully disclose it to investors. The

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[1] Similar incidents include (i) a false statement in 2022 that Eli Lilly and Co. intended to offer its insulin at no cost (Barr 2022; Harwell 2022) and (ii) incorrect information disseminated through fake Twitter accounts in 2013 about Audience Inc. and Sarepta Therapeutics Inc. (SEC 2015). For discussions of rumors and false information, see Schindler (2007); Marett and Joshi (2009); Kapferer (2013); Kimmel (2013); Ahern and Sosyura (2015); Kohlbeck and Vakilzadeh (2020); Alperovych, Cumming, Czellar, and Groh (2021); Davis, Khadivar, and Walker (2021); Cai, Quan, and Zhu (2023).

manager's objective is to maximize her firm's (expected) market price. Investors evaluate the firm after considering the manager's disclosure and news (a signal) from an external source with uncertain and unobservable precision. In the main part of the paper, we focus on the case of early disclosure, where the manager decides whether to reveal her private information before the external public news.[2]

As in prior research, the firm's disclosure strategy follows a threshold rule: high values are disclosed, while low ones are not. However, in our model, the presence of external news complicates the manager's decision-making. On one hand, if the anticipated external news turns out to be good, the manager may regret having disclosed prematurely. On the other, if the news is bad, the manager may regret remaining silent. Ultimately, when deciding whether to disclose, the manager compares the disclosure price with the expected nondisclosure price. The former perfectly reflects the observed and disclosed firm value, while the latter depends critically on how investors assess the reliability of the external news in conjunction with the firm's silence. We show that this assessment affects asset pricing in unexpected ways.

One of our novel results is that the equilibrium nondisclosure price exhibits a nonmonotonic relationship with the external news (and reacts to it asymmetrically). This relationship is positive for very good or very bad external news, as one would expect. However, in the intermediate region, better news may paradoxically lead to lower market prices. This decrease is driven by the investors' (endogenous) updating of their beliefs about the manager's information endowment and the precision of external news, absent a firm disclosure.

To elaborate, even though the manager's information endowment and the news' precision are assumed to be independent, we find that investors' beliefs about them are *endogenously intertwined*, since both are updated jointly following nondisclosure. Specifically, relatively bad external news strengthens the joint beliefs that the news is precise and that the nondisclosing manager is informed; relatively good news has the opposite effect. This pattern is

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[2] We explore delayed disclosure in an extension. Early disclosure can arise because the firm's conference call is scheduled before a media broadcast or a scheduled release of a peer's financial report or restatement, or because the manager can only disclose before the so-called quiet period in the lead-up to an initial public offering (IPO) or the closing of a business quarter.

driven by the manager's strategic behavior: since she discloses favorable information and withholds unfavorable, investors understand that an informed manager who did not disclose must have observed unfavorable information. Thus, when faced with a silent manager, the investors perceive relatively bad news as more precise, which in turn strengthens their beliefs that the silent manager is informed. Conversely, after observing good news, the investors are less likely to believe that the news is precise and that the silent manager is informed. Thus, the market discounts good external news.

A key implication of our argument is that the market price exhibits a sharp decline in the region where investors expect the manager to begin disclosing. Outside this region, the price responds conventionally to external news—rising when news is better—resulting in an overall *nonmonotonic* price pattern. Nevertheless, the firm's disclosure follows a threshold rule, and equilibrium is maintained due to a critical property: at the point where the disclosure and expected nondisclosure prices intersect, the latter declines steeply and remains below the corresponding firm value. This dynamic makes disclosure of any value beyond this point even more attractive. Additionally, before its sharp decline, the expected nondisclosure price remains higher than low firm values, incentivizing the manager to remain silent.

Another implication of grain-of-salt belief updating is that the nondisclosure price reacts more strongly to bad external news than to good news. This asymmetry arises because, when managers remain silent, investors perceive bad news as more credible, whereas they are more skeptical of good news. Furthermore, comparing disclosure models with and without external news, we find that the presence of external news discourages early disclosures, with this crowding-out effect strengthening when the news is more likely to be precise. The explanation for this effect is nuanced. On average, firms choosing not to disclose have weaker fundamentals. As a result, they benefit from external news that is imperfectly precise, particularly when it does not directly reflect their fundamentals but still leads to a higher valuation, which allows them to piggyback on the external news. As external news becomes more precise, investors place greater weight on it, amplifying this benefit. However, a counterveiling effect also emerges: when external news becomes more precise, the probability of firms gaining from it diminishes. We show that the first effect dominates, and so greater precision ultimately crowds out firm disclosures.

We extend our model to a setting in which the manager can delay disclosure. Absent additional frictions, the manager derives no strategic advantage from disclosing early, as she could always wait to observe the external news before making any decisions. In such case, the positivity of the external news plays a critical role in the disclosure decision. Good news discourages delayed disclosure, whereas bad news encourages it. This effect strengthens as the external news becomes more likely to be precise. Overall, the mere presence of external news crowds out (crowds in) late corporate disclosures when firm values are more likely to be high (low) in the economy.

In a frictionless world, when only late disclosure arises in equilibrium, the nondisclosure price becomes monotonic, although it remains more sensitive to bad news than to good news. However, we find that nonmonotonicity can reemerge when scheduling costs affect disclosure timing. These costs may stem from timeline constraints, reputational damage due to withholding information, cancellation of previously scheduled announcements, disruptions to internal planning, decision-making processes, and investor relations activities. When these costs make both early and late disclosure possible, the nondisclosure price exhibits nonmonotonicity around the value that the market expects to be disclosed early.

Our model generates several empirical predictions. First, when the precision of external news is uncertain, the stock prices of nondisclosing firms may exhibit a nonmonotonic response. While very good or very bad external news leads to expected price movements, intermediate news may paradoxically result in lower prices. Second, we predict an asymmetry in market reactions to external news: stock prices of nondisclosing firms will be more sensitive to bad news than to good news when investors are uncertain about the news's precision. Third, external news influences firms' disclosure probabilities in distinct ways. Firms are less likely to disclose before entering a quiet period when external news is present. They are more (less) likely to disclose in response to bad (good) external news, with this effect intensifying when the news is more likely to be precise. Overall, we predict that firms are less likely to reveal information in response to external news in industries with relatively high firm values, and vice versa. Fourth, volatility will be higher following bad external news than good news, with the volatility gap widening when the news is more precise. Finally, we show that firms may strategically withhold disclosures when external news is bad but selectively release disclosures that are slightly worse than the external news when it is good. Together, our predictions offer a novel perspective on how firms respond to external news, particularly in environments where the credibility of third-party sources—such as social media, ESG ratings, and regulatory disclosures—is uncertain.

2 Literature Review

This paper contributes to the literature on voluntary disclosure when the market is uncertain about whether firms possess private information.[3] Several studies examine models where, in addition to corporate disclosure, there is external news.[4] A tenet in this literature is that the arrival of external news is correlated with the firm's information endowment, assuming the external news is completely imprecise (as in Dye and Sridhar 1995) or perfectly precise (as in Frenkel, Guttman, and Kremer 2020). In these studies, the market updates its beliefs about the manager's information endowment because of the assumed exogenous correlation. Without such a correlation, there would be no effect on beliefs. In Frenkel, Guttman, and Kremer (2024), it is not the arrival but the content of the external signal that is correlated with the manager's information endowment, and the resulting equilibrium necessarily in-

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[3] The voluntary disclosure literature traces its origins to the seminal contributions of Grossman (1981) and Milgrom (1981). Uncertain information endowment (Dye 1985) is one of the frictions that prevents full information unraveling. In the absence of known frictions, Einhorn (2018) shows that unraveling may also be inhibited by the presence of competing sources of information.

[4] Related, Bloomfield, Heinle, and Luneva (2024) and Ebert, Schäfer, and Schneider (2022) analyze models where the firm's information may be leaked. The anticipation of such leaks affects the manager's disclosure decision.

volves mixed strategies.[5] Across all of these studies, market prices increase *monotonically* with the external signal.

By contrast, we study a model in which both the arrival and the content of external news are independent of whether the manager is privately informed, while the signal's precision remains uncertain.[6] Under these conditions, we show that pure-strategy equilibria arise and that the resulting belief updating generates complex price dynamics. Most notably, we demonstrate that, when managers remain silent, market prices can be *nonmonotonic* in the favorability of external news—a finding that contrasts with theoretical models of disclosure and helps explain mixed empirical results.[7]

Broadly speaking, our work also relates to a stream of literature (e.g., Acharya, DeMarzo, and Kremer 2011; Menon 2020) in which the firm value and the external signal are imperfectly and positively correlated but (unlike in our model) the correlation is *known and fixed*. Thus, while the arrival of the signal affects the (conditional) distribution of the firm value, investors' beliefs about the correlation remain constant, regardless of the signal realization. As a result, market prices in these models are *monotonic* (see Section 5 for discussion). By contrast, we study a setting where investors are uncertain about the correlation between external news and firm value, i.e., about the signal's precision. The arrival of the signal in our model leads to a grain-of-salt pattern of belief updating and, as a result, *nonmonotonicity* in market price. To our knowledge, this result is a novel implication of the form of the endogenous updating that we study.

Few studies also consider uncertain precision. For example, Heinle and Smith (2017) considers a firm's mandatory disclosure of the unobservable by the market variance of its cash flows. Subramanyam (1996) studies the effect of uncertainty regarding the precision

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[5] Specifically, their model assumes that the external signal equals the true firm value with positive probability only if the manager is informed; otherwise, the signal is pure noise.

[6] Such settings are common in practice—for instance, the credibility of mainstream or social media coverage is often ambiguous and unrelated to the firm's internal information.

[7] For example, Capkun, Lou, Otto, and Wang (2023), Breuer, Hombach, and Müller (2022), and Baginski and Hinson (2016) find a negative relationship between voluntary disclosure and external information, whereas Seo (2021) reports a positive relationship. Sletten (2012) shows that firms may respond to external news by disclosing even unfavorable information.

of information on prices when firms do not disclose information. Under the distributional assumptions of Subramanyam (1996), extreme signals are perceived as more likely to be the result of noise; consequently, stock prices may not be monotonic. We also find that prices may be nonmonotonic, but for a fundamentally different reason, namely the managers' strategic behavior in the presence of uncertain external news. Importantly, the nonmonotonicity in our model arises under any distributional assumption and occurs around the disclosure threshold (which is typically intermediate), while Subramanyam (1996) argues that nonmonotonicity may arise under specific distributions in response to extreme signals. (For extensive discussion of the driving forces, see Section 5.)[8]

As we do, several researchers find that market prices react to news asymmetrically. However, the economic impetus for the results differ. In Veronesi (1999), markets react asymmetrically because risk-averse investors hedge against changes in the fundamental and overreact (underreact) to unfavorable (favorable) news. In Banerjee and Green (2015), asymmetric price reactions emerge because the traders have mean-variance preferences: good news reduces the expected fundamental value and amplifies the negative risk effect, whereas bad news increases it and counteracts the risk.

3 Model

We extend the voluntary disclosure model with uncertain information endowment (Dye 1985) by incorporating external news with uncertain precision.

Players. The model entails a manager (she) who is a price-maximizer and runs a firm with value $v \in V \equiv [v_{min}, v_{max}]$, where $-\infty < v_{min} < v_{max} < \infty$. A group of risk-neutral investors observes all publicly available information Ω, forms beliefs, and prices the firm at

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[8] If the uncertainty about the external news is extreme, the market price in our model becomes discontinuous. The price in Bond, Goldstein, and Prescott (2010) is also discontinuous but this is due to firm's actions after inferring information reflected in stock prices. In their model, unlike ours, a jump upwards can occur. Prices can also be discontinuous due to exogenous insurance of investment portfolio (Gennotte and Leland 1990) or presence of uninformed investors who can buy at relatively high prices and endogenously act as insurance (Barlevy and Veronesi 2003).

$P(\Omega) = \mathbb{E}[v|\Omega]$. We assume that the price is set once after Ω is realized.[9]

Information structure. The common prior belief is that v is drawn from a cumulative differentiable distribution function G, which we assume is log-concave with prior expectation $\mathbb{E}[v] = \mu$.[10] The publicly available information Ω consists of an external signal and the manager's voluntary disclosure:

1. *External signal.* There is a public signal $s \in V$ from an external nonstrategic source. Throughout the manuscript, we refer to s as external news, with relatively high (low) realizations of s interpreted as relatively good (bad) news. The distribution of s *conditional* on a realized firm value is $F(s|v, \phi)$.[11] We assume that F belongs to the log-concave family.[12] The parameter $\phi \in [0, 1]$ reflects how much information s provides about v, with higher values of ϕ implying that the signal provides more information. To articulate this notion more clearly, we will refer to ϕ as the correlation between s and v. When $\phi = 1$, we say that s is perfectly precise—formally, $v|s$ converges in distribution to a degenerate point mass on s (that is, $s = v$), which implies that $F(s|v, \phi = 1) = G(s)$. When $\phi = 0$, we say that s is completely imprecise—formally, $v|s$ converges in distribution to an independent draw of $v \sim G$ for all s. For tractability reasons, we impose that $F(s|v, \phi = 0) = G(s)$. When $\phi \in (0, 1)$ the signal is imperfectly correlated with the firm value and is thus imperfectly precise.[13,14]

Neither the investors nor the manager observe ϕ, and therefore the precision of

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[9] In a previous version (Libgober, Michaeli, and Wiedman, 2023) of this paper, we showed that frequently adjusted prices do not qualitatively change our main results.

[10] The log-concave family includes multiple common distributions: normal, uniform, exponential, chi, beta, gamma, logistic, etc. Although many of our results do not require log-concavity, we maintain this assumption throughout.

[11] The source of the signal is nonstrategic.

[12] Formally, we require $F(s|\cdot, \phi)$ and $F(\cdot|v, \phi)$ to both be log-concave functions for all ϕ.

[13] To see this, consider the following illustrative example. Suppose that the signal is $s = \alpha v + (1 - \alpha)u$, where $\alpha \in [0, 1]$ is unknown, $v \sim \mathcal{U}[v_{min}, v_{max}]$ and $u \sim \mathcal{U}[v_{min}, v_{max}]$. Then, when $\alpha = 1$, we have $s = v$, $\phi = 1$ and $F(s|v, \phi = 1) = G(s)$ is uniform. On the other hand, when $\alpha = 0$, we have $s = u$, $\phi = 0$ and $F(s|v, \phi = 0) = G(s)$ is uniform. For any $\alpha \in (0, 1)$, it holds that $\phi \in (0, 1)$ and the distribution $F(s|v, \phi)$ is triangular rather than uniform.

[14] We assume that all distributions and densities are continuous in ϕ and smooth in s and v, for all $\phi < 1$. Also, for every s in the support, the marginal distribution over s given ϕ has a positive density.

the signal is uncertain. For simplicity, we posit that ϕ can assume two values, $\underline{\phi}$ and $\overline{\phi}$, such that $0 \leq \underline{\phi} \leq \overline{\phi} \leq 1$. The common prior belief is that $\phi = \overline{\phi}$ (i.e., the signal is more precise) with probability $q \in (0, 1)$ and $\phi = \underline{\phi}$ (i.e., the signal is less precise) with probability $1 - q$. The uncertainty about ϕ is jointly reflected by q and $\Delta_\phi \equiv \overline{\phi} - \underline{\phi} \in [0, 1]$. When either $q = \{0, 1\}$ or $\Delta_\phi = 0$ (so that $\underline{\phi} = \overline{\phi}$) the precision is certain—in Section 5 we briefly consider the outcome in such a setting for comparison with our results.

2. *Manager's information and voluntary disclosure.* The manager is privately informed about v with probability $p \in (0, 1)$, in which case we say she is informed ($\kappa = I$); with the remaining probability, she is uninformed ($\kappa = U$). The information endowment κ is independent of v and ϕ, and is privately known to the manager but unobservable to investors. We refer to a manager informed about relatively high (low) v as having relatively favorable (unfavorable) information.

   An uninformed manager cannot credibly communicate her lack of information and must remain silent ($d = \varnothing$). An informed manager may choose to voluntarily disclose the firm value ($d = v$) to investors before the arrival of an external signal—what we refer to as early disclosure.[15] In the extension of Section 6, we allow the manager to delay disclosure until after the signal is realized. As is standard in the voluntary disclosure literature, any disclosed value is verifiable and hence truthful. We also assume that, in equilibrium, a manager who is indifferent about disclosure chooses to remain silent. Our analysis focuses on pure strategies.

   As is well-known (Dye 1985), when the manager's information endowment is uncertain and there is no external signal—or equivalently when the signal is perfectly imprecise—then the manager discloses all observed v that exceed a certain threshold.

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[15] Early disclosure could arise because the manager is slated to hold a conference call ahead of a media broadcast or an anticipated release of a peer's financial statements or a restatement. Similarly, external news may be expected to arrive during a quiet period in the lead-up to an IPO or quarter close—under such circumstances, disclosing after the news would violate quiet-period rules.

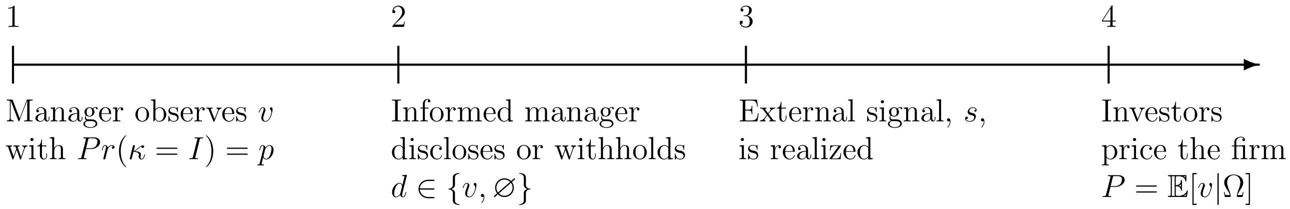

Figure 1: Timeline of events

We denote this threshold v^D and use it as a reference point.

Timeline. On date 1, the manager observes the firm value v with probability p. On date 2, the manager can disclose her information if she is informed. On date 3, the external signal s is realized. On date 4, the investors price the firm.

4 Main Analysis

We now proceed with the model's analysis and the key insights from our paper: the emergence of a nonmonotonic price when the manager remains silent.

Our solution concept is backward induction. The first step is to consider investor beliefs and market price on date 4 for a given signal realization and manager's disclosure or nondisclosure. For now, we conjecture but later formally verify that the disclosure follows a threshold rule, whereby the manager withholds values that fall below some—*fixed at that point in time*—threshold $\widehat{v} \in V$ and discloses otherwise. The market price on date 4, conditional on the manager's disclosure, is determined solely by the disclosed value, $P(s, v) = \mathbb{E}[v|\overline{\phi}, s] = v$. That is, investors disregard the external signal following disclosure because the manager observes the firm value precisely and her disclosure is truthful. Conversely, the date-4 price, conditional on the manager's silence, can be expressed as

$$P(s, \varnothing) = \mathbb{E}[v|s, \varnothing] = \sum_{\kappa,\phi} \Pr(\kappa, \phi|s, \varnothing) \cdot \mathbb{E}[v|\kappa, \phi, s, \varnothing]. \tag{1}$$

Lemma 1. *Fix a threshold $\widehat{v} \in V$. There exist $\bar{\Delta} \in (0,1)$, s', s'' with $s' < \widehat{v} < s''$ such that:*

(i) If $f(\cdot|\cdot,\phi)$ satisfies the strict monotone likelihood ratio property (strict MLRP), then $\mathbb{E}[v|\kappa, \phi, s = s'', \varnothing] > \mathbb{E}[v|\kappa, \phi, s = s', \varnothing]$ for any ϕ and κ.

(ii) If $\Delta_\phi > \bar{\Delta}$, the following inequalities hold:

$$\begin{aligned} \Pr(\kappa, \phi|s', \varnothing) &> \Pr(\kappa, \phi|s'', \varnothing) \text{ for } \phi = \overline{\phi} \text{ and } \kappa = I; \\ \Pr(\kappa, \phi|s', \varnothing) &< \Pr(\kappa, \phi|s'', \varnothing) \text{ otherwise.} \end{aligned}$$

Part (i) of Lemma 1 shows that (under a technical condition satisfied for a large family of distributions) the term $\mathbb{E}[v|\kappa, \phi, s, \varnothing]$ is increasing in s. Simply put, given the manager's silence, better news leads to a higher posterior expectation for given ϕ and κ. Part (ii) implies that investor beliefs about the occurrence of ϕ and κ can be either increasing or decreasing in s. Notably, the market assigns a *lower* likelihood to the event $(\phi = \overline{\phi}, \kappa = I)$ when the external signal is higher (above the threshold $\widehat{v}$) and the precision of the signal is highly uncertain; see Figure 2. To understand the intuition, consider investors who believe that the manager is informed and did not disclose because she observed unfavorable information (low v). Observing bad external news (low s) then reinforces investors' belief that the news is precise. However, observing good news (high s) contradicts the manager having unfavorable information and as a result weakens investors' belief that the signal is precise.

To see the above point more clearly, suppose that $\overline{\phi} = 1$ and $\underline{\phi} = 0$, i.e., the news is either perfectly precise or perfectly imprecise. In an equilibrium with disclosure threshold $\widehat{v}$, investors infer that an informed manager who did not disclose must have observed $v \leq \widehat{v}$.[16]

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[16] We formally prove the existence of unique threshold equilibrium below.

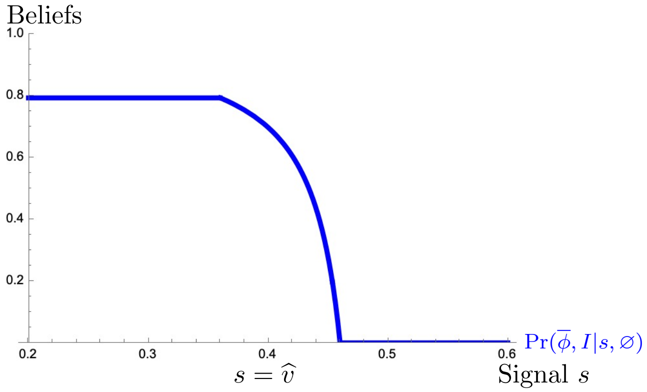

Figure 2: Joint investor beliefs about the probability that $\phi = \overline{\phi}$ and $\kappa = I$

Numerical example where $\widehat{v} = 0.4$, $p = 0.9$, $q = 0.7$, and the signal is either $s = \frac{1}{5}v + \frac{4}{5}u$ (corresponding to $\phi = \underline{\phi} > 0$) or $s = \frac{9}{10}v + \frac{1}{10}u$ (corresponding to $\phi = \overline{\phi} < 1$) with $v \sim \mathcal{U}[0, 1]$ and $u \sim \mathcal{U}[0, 1]$

Thus, if the external signal exceeds the disclosure threshold ($s > \widehat{v}$), it will be rationally inconsistent for the investors to believe that the signal is precise and that the manager is informed at the same time. Consequently, they infer that $\Pr(I, \overline{\phi}|s'', \varnothing) = 0$. By contrast, investors perceive a signal below the disclosure threshold ($s \leq \widehat{v}$) as more likely to be precise, which strengthens their joint beliefs that the nondisclosing manager is also informed—in this case, $\Pr(I, \overline{\phi}|s', \varnothing) > 0$.[17] To summarize, bad s strengthens the beliefs of investors that the external news is precise and that the nondisclosing manager is informed. Conversely, good s weakens investors' beliefs: such news is less likely to precisely describe the value of a firm run by an informed, nondisclosing manager.[18]

Lemma 1 considered an arbitrary threshold $\widehat{v}$ and did not characterize equilibrium behavior. To study the equilibrium outcome, one needs to bear in mind that the manager's disclosure decision is made *before* the signal realization s and *without* knowing the signal's correlation ϕ. Thus, when deciding on date 2 whether to reveal an observed firm value, the

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[17] One complication that emerges when $\underline{\phi} > 0$ and $\overline{\phi} < 1$ is that $\Pr(\kappa, \phi|s, \varnothing)$ is relatively unrestricted, precluding obtaining comparative statics of $\Pr(\phi, \kappa|s, \varnothing)$ in s which hold for general $F(s|v, \phi)$. Nevertheless, our proof verifies that the above intuition is maintained whenever Δ_ϕ is sufficiently large.

[18] The nontrivial effect of s on beliefs arises *only* in the presence of strategic disclosure. To see why, note that, if the manager cannot disclose for exogenous reasons (or equivalently has no information with certainty, $p = 0$), the lack of disclosure and the signal realization carry no information about ϕ and κ, and so beliefs in this scenario remain constant: $\Pr(\phi, \kappa|s, \varnothing) = \Pr(\kappa|\varnothing) \Pr(\phi|s) = \Pr(\kappa) \Pr(\phi)$, for any s, ϕ and κ.

price-maximizing manager compares the disclosure price, $P(s, v) = v$ with her *expectation* of the nondisclosure price in (1) over all possible signal realizations and correlations. Note that Lemma 1 implies the price might not be monotonic in s and an informed manager's expectation of s depends on the observed firm value—thus, the manager's expectation of the nondisclosure price might not be monotonic in v. As a result, the existence and uniqueness of an equilibrium are not guaranteed.[19] To show these outcomes, we impose the following (technical) assumption for the function $h(v|\phi, s) \equiv \frac{f(s|v,\phi)g(v)}{\int_{v_{min}}^{v_{max}} f(s|v,\phi)g(v)dv}$ defined to be the probability distribution of v conditional on the signal s and correlation ϕ.

Assumption 1. *It holds that* $\lim\limits_{\underline{\phi}\to 0} \max\limits_{v,s} |\frac{\partial}{\partial s} h(v|s, \underline{\phi})| = 0$ *and* $\lim\limits_{\underline{\phi}\to 0} \max\limits_{s,v} |\frac{\partial}{\partial v} f(s|v, \underline{\phi})| = 0$.

In words, when the signal is very imprecise, the distributions relating signals to values become unresponsive to each other. Intuitively, Assumption 1 ensures that the manager's expected nondisclosure payoff cannot increase too steeply in her value.[20]

Lemma 2. *Suppose that* $\overline{\phi} = 1$ *and Assumption 1 holds.*

(i) There exists $\bar{\Delta} < 1$ *such that when* $\Delta_\phi \geq \bar{\Delta}$ *a threshold equilibrium exists where the manager discloses all values exceeding a threshold* $v^E \in (v^D, \mu)$ *and withholds otherwise.*

(ii) If $\Delta_\phi = 1$*, the equilibrium described in part (i) is unique.*

Despite the challenges described above, an equilibrium still exists due to a key property: at the point where disclosure and expected nondisclosure prices meet, the latter steeply declines

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[19] To understand the subtleties behind the existence and uniqueness of threshold equilibria, it is instructive to consider $q = 0$ and $\underline{\phi} = 0$, so that the external signal is pure noise with certainty. In this case, if the manager does not disclose, her payoff is $\mathbb{E}[v|v \leq \widehat{v}]$. Crucially, this payoff does not depend on what the manager's actual value is as nondisclosure always gives the same payoff. But this is not true in our model, since informed managers with different firm values expect different distributions over s (because s is correlated with v). As a result, $\mathbb{E}[P(s, \varnothing)|v]$ depends on v. How this expectation changes depends not only on the slope of the price function but also on how its expectation changes in s. Without the restrictions we have imposed, this property may prevent the existence of threshold equilibria, as some other structure is needed to ensure that the gain from nondisclosure does not increase much quicker than the gain from disclosure.

[20] To explain why this assumption is necessary, we note that a technical complication in our model is that the manager's nondisclosure payoff is nonconstant in the firm value. In some cases, an increase in v by an amount x may lead to an increase in the expected nondisclosure payoff by more than x. In principle, this challenge could prevent the existence of a threshold equilibrium, since, if it were to hold, then, for a given threshold, an increase in the manager's value may lead to her preferring to remain silent.

and remains below the firm value, making disclosure of any value beyond this realization even more attractive. Furthermore, the expected nondisclosure price exceeds (low) firm values in the region before the steep decline—hence, in this region, the manager prefers to silence.

In addition to establishing the existence and uniqueness of a threshold equilibrium, Lemma 2 characterizes the set of managers who disclose: Managers who withhold information when there is no external news (i.e., those with value $v < v^D$) continue to do so when there is external news. In addition, managers observing $v \in [v^D, v^E]$ also withhold their information, as they benefit from the market forming its expectation of firm value based only on the external news. This benefit is linked to the investors' uncertainty about the precision of the external news and the manager's information endowment. Specifically, when investors suspect $\phi = \underline{\phi}$, they assign a higher likelihood that the manager is uninformed ($\kappa = U$) and put less weight on s and more on their prior expectation. This allows managers with $v < v^E$ to pretend to be uninformed.

To establish whether external news crowds in or crowds out disclosure, we compare the probability of disclosure with and without external news. Overall, the probability of a firm's disclosure is $\Pr(d = v) = \Pr(\kappa = I)\Pr(d = v|\kappa = I) = p \times (1 - G(\widehat{v}))$ for disclosure threshold $\widehat{v} = \{v^E, v^D\}$. Our finding that $v^E > v^D$ implies that external news crowds out firm disclosures that precede the news. The explanation for this effect is nuanced. Firms that choose not to disclose tend to have weaker fundamentals. As a result, they benefit from external news that is imperfectly precise, particularly when it does not directly reflect their fundamentals and leads to a higher valuation—allowing them to piggyback on the external news. As the news become more precise, investors place greater weight on it, amplifying this benefit. However, a counterweight emerges: when external news becomes more precise, the probability of firms gaining from it diminishes. We show that the first effect dominates, implying that greater precision ultimately crowds out firms' disclosures.

Parts (i) and (ii) of Lemma 1 jointly implied that the properties of the market price in (1) are not straightforward for an arbitrary threshold. Having established the existence

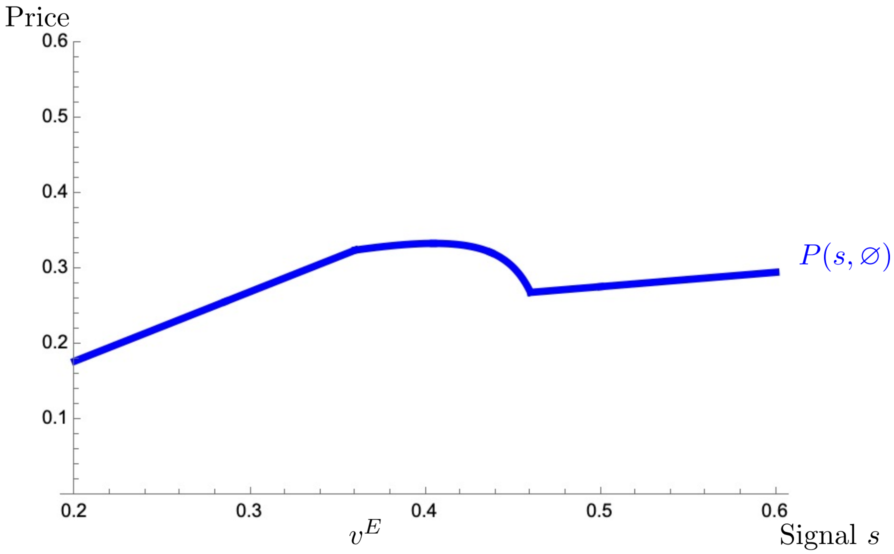

Figure 3: Market price in equilibrium

Numerical example where $p = 0.9$, $q = 0.7$, and the signal is either $s = \frac{1}{5}v + \frac{4}{5}u$ (corresponding to $\phi = \underline{\phi} > 0$) or $s = \frac{9}{10}v + \frac{1}{10}u$ (corresponding to $\phi = \overline{\phi} < 1$) with $v \sim \mathcal{U}[0,1]$ and $u \sim \mathcal{U}[0,1]$

of an equilibrium threshold v^E in Lemma 2, we are now ready to go back to the date-4 nondisclosure market price and discuss its shape in equilibrium.

Proposition 1. *Suppose that* $\overline{\phi} = 1$ *and Assumption 1 holds. For* $\Delta_\phi \geq \bar{\Delta}$ *as defined in Lemma 2, there exists* $\delta > 0$*,* s' *and* s'' *with* $s' < v^E < s''$ *such that the nondisclosure price is nonmonotonic in* s *around the equilibrium disclosure threshold* v^E *in the sense that*

$$\begin{aligned} P(s'-\delta,\varnothing) &< P(s',\varnothing) \\ P(s',\varnothing) &> P(s'',\varnothing) \\ P(s'',\varnothing) &< P(s''+\delta,\varnothing). \end{aligned}$$

The nonmonotonicity result in Proposition 1—illustrated in Figure 3—emerges due to the investors' belief updating and the manager's disclosure behavior. To gain intuition, recall that neither the investors nor the manager know how reliable the external news is. Firm disclosure—which constitutes a truthful revelation of perfect information—makes considerations of the reliability and content of the external news irrelevant (and fully resolves uncertainty about whether and what the manager knows). The disclosure price then simply reflects the manager's information. But when the manager remains silent, investors' beliefs about the news' precision (ϕ) and the manager's information endowment (κ) really matter for the price. Because the manager tends to disclose high v and withhold low v, a lack of disclosure and good external news suggest that the external news is not so precise (i.e., it should be taken with a grain of salt) and the firm value is worse than the signal suggests. Thus, better external news can paradoxically lead to a lower market price. That is, the nondisclosure price exhibits a steep decline in the neighborhood of the disclosure threshold. Outside of this region, prices behave predictably (i.e., increasing in s). This implies that the nondisclosure price is *nonmonotonic* in the positivity of external news.

In addition to nonmonotonicity, grain-of-salt updating has another implication: nondisclosure prices react asymmetrically to external news.

Proposition 2. *Suppose that* $\overline{\phi} = 1$ *and Assumption 1 holds. Then, for* $s' < v^E < s''$ *as defined in Proposition 1, there exists* $\widetilde{\Delta}$ *such that whenever* $\Delta_\phi > \widetilde{\Delta}$*, it holds that* $\frac{\partial P(s,\varnothing|s<s')}{\partial s} > \frac{\partial P(s,\varnothing|s>s'')}{\partial s} > 0$.

Simply put, when firms do not disclose, investors are more sensitive to bad news than to good news. The intuition, again, is that, if good news is more precise about v, firms are more likely to disclose. Managers' silence thus implies that good external news is less likely to be precise—and rational investors react less sensitively. Conversely, bad news is perceived as more likely precise and leads to a stronger price reaction. Figure 3 illustrates this asymmetry, in addition to the previously described nonmonotonicity.

5 Discussion of Price Nonmonotonicity

The preceding section established that, amid firms' silence, market prices are nonmonotonic.[21] The potential for nonmonotonicity arises because of the rational way in which the market infers news' precision and the manager's information endowment (Lemma 1). To

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[21] Nondisclosure prices are not only nonmonotonic but also discontinuous when $(\underline{\phi}, \overline{\phi}) = (0, 1)$. This emerges because the CDF of s, conditional on $\phi = \overline{\phi}$ is discontinuous at $s = v$. Less extreme specifications of uncertain precision smooth out the discrete drop of the price function.

appreciate the importance of this updating, consider naive investors, who, regardless of the news, fix their joint beliefs at some value, say $\varphi(\phi, \kappa) = \Pr(\phi)\Pr(\kappa|\varnothing)$. Then the nondisclosure price in (1) would simply be given by $\varphi(\overline{\phi}, U)\cdot s + \varphi(\overline{\phi}, I)\cdot s + \varphi(\underline{\phi}, U)\cdot \mu + \varphi(\underline{\phi}, I)\cdot \mathbb{E}[v|v \leq \widehat{v}]$, which is monotonic increasing in s.

For the novel force—complex updating of beliefs—identified in our model to result in nonmonotonicity, two model ingredients must be present: (i) uncertainty about the precision of the external news and (ii) the ability of firms with uncertain information endowment to voluntarily disclose. To illustrate the importance of these ingredients, we now briefly discuss simplified versions of the model.

Model with certain precision of the news. To begin, consider a simplified version of our model (superscript C) where the correlation of the signal with the firm value is certain, i.e., either $\Delta_\phi = 0$ or $q \in \{0, 1\}$. It is easy to see that there exists a unique threshold $v^C \in V$ such that the manager discloses if $v > v^C$ and withholds otherwise. Like before, the price following disclosure is $P(v) = v$. By Milgrom (1981), the price following nondisclosure, $P(s, \varnothing)^C = \mathbb{E}[v|\varnothing, \phi, s]$, is monotonic increasing in s, whenever the conditional distribution $F(\cdot|\cdot, \phi)$ satisfies the strict monotone likelihood ratio property (MLRP), $\frac{f(s|v,\phi)}{f(s|v',\phi)} > \frac{f(s'|v,\phi)}{f(s'|v',\phi)}$ for $s > s'$ and $v > v'$.[22] Examples of disclosure models with certain precision of external news where MLRP holds are Dye and Sridhar (1995); Menon (2020); Frenkel, Guttman, and Kremer (2020). In these studies, equilibrium market prices are monotonic increasing in the signal. By contrast, the results in Section 4 of this paper imply that when the precision of the external news is sufficiently uncertain (high Δ_ϕ), the nondisclosure price is nonmonotonic *even if* $F(\cdot|\cdot, \phi)$ satisfies MLRP.

Model without voluntary disclosure. Next we consider a simplified setting (super-

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[22] The MLRP property is satisfied for many common specifications of F; for instance, if $s = v + \varepsilon$, where ε is an independent random variable with log-concave density. Milgrom (1981) showed MLRP implies that the posterior distribution of v given s is increasing in first-order stochastic dominance (FOSD), for any prior over v. In our context, this result immediately implies that, for any equilibrium distribution over v conditional on nondisclosure, the posterior expectation of v given s is increasing in s (using the observation that an increase in the distribution over a random variable in the FOSD order implies an increase in its posterior expectation). For completeness, the appendix presents a proof of the claim that, if $s = v + \varepsilon$ for ε with log-concave density, then the posterior expectation is increasing in the signal for every prior.

script N) where the firm cannot disclose (e.g., due to a quiet period) or has no information with certainty ($p = 0$). Then the market price is simply given by

$$P(s)^N \quad = \quad \Pr(\phi = \underline{\phi}|s) \cdot \mathbb{E}[v|\phi = \underline{\phi}, s] + (1 - \Pr(\phi = \underline{\phi}|s)) \cdot \mathbb{E}[v|\phi = \overline{\phi}, s]. \tag{2}$$

It is easy to see that the price in (2) is monotonically increasing if two conditions are satisfied:

(i) The conditional distribution $F(\cdot|\cdot, \phi)$ satisfies MLRP so that the conditional expectation $\mathbb{E}[v|\phi, s]$ is monotonically nondecreasing in s for any $\phi \in [0, 1]$ (Milgrom 1981).

(ii) It holds that $\int_{v_{min}}^{v_{max}} f(s|v, \phi)g(v)dv$ is the same for all ϕ so that the conditional probability $\Pr(\phi = \underline{\phi}|s) \in [0, 1]$ is independent of s.

The analysis in Section 4 shows that allowing for voluntary disclosures by a firm with uncertain information endowment and accounting for the strategic behavior of firms leads to price nonmonotonicity near the disclosure cutoff, *even if* F is such that conditions (i) and (ii) are satisfied and the price would otherwise be monotonic absent the ability to disclose.

What happens when either condition (i) or condition (ii) are not satisfied? One such example is Subramanyam (1996), where there is no disclosure and the signal is the sum of the firm value and noise. Both firm value and noise are normally distributed. The signal's precision (in this case, represented by inverse of variance) is uncertain, and the market updates beliefs about it after observing s. That is, Subramanyam (1996) studies variance uncertainty, which is a special case of the correlation uncertainty we study.[23] In the case of Subramanyam (1996), variance uncertainty can imply nonmonotonicity even in the absence of disclosure

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[23] In Subramanyam (1996), there is no disclosure, and the signal is $s = v + \varepsilon$ for $v \sim \mathcal{N}(m, 1/\nu)$ and $\varepsilon \sim \mathcal{N}(0, 1/n)$, so that s has mean m and precision $w = \frac{\nu n}{\nu + n}$. The precision w is uncertain and the market updates beliefs about it after observing s. To see that this is a special case of correlation uncertainty, compute ϕ, assuming that s is generated as in Subramanyam (1996): $\phi = corr(v, s) = \frac{cov(v,s)}{\sqrt{var(v)var(s)}} = \frac{cov(v,v+\varepsilon)}{\sqrt{(1/\nu)(1/w)}} = \frac{1/\nu}{\sqrt{(1/\nu)(1/w)}} = \sqrt{\frac{w}{\nu}} \in (0, 1)$. Thus, uncertainty over w translates into uncertainty over ϕ: in cases where the signal is very noisy, the correlation is also very low, and vice versa. However, correlation uncertainty is *not* a special case of variance uncertainty. This can be seen by considering the case where $\phi \to 0$. With variance uncertainty, this would imply $w \to 0$, so that the variance of the signal also approaches infinity. By contrast, our formulation allows for the variance of the signal to be constant in ϕ.

because condition (ii) is not satisfied. However, this nonmonotonicity will typically be observed for extreme realizations—the intuition is that extremely large or extremely small signal realizations are less likely to be precise. By contrast, we identify nonmonotonicity *in the neighborhood of the disclosure threshold*, which will typically be intermediate. Importantly, as we pointed out above, unlike Subramanyam (1996) the nonmonotonicity in our model arises even when condition (ii) is satisfied so that $\Pr(\phi|s)$ is constant in s.[24]

6 Extension With Option to Delay

We now extend our results by allowing the manager to endogenously decide *when* to disclose—either before (as our previous analysis considered) or after the signal s is realized. We assume that delaying the disclosure comes at scheduling cost $c_L \geq 0$ (e.g., due to reputation loss for withholding the information early on, cancellation of previously scheduled announcement, or disruption of internal planning, decision-making processes and investor relations activities), whereas advancing it to an earlier date comes at scheduling cost $c_E \geq 0$ (e.g., due to schedule constraints, reduced preparation time, reallocation of resources and personnel to meet accelerated timeline). The superscript "L" denotes late and "E" denotes early. To highlight the intuition for our results, in this extension, we focus on the case where $\underline{\phi} = 0$ and $\overline{\phi} = 1$, i.e., the signal is either perfectly precise (truth) or perfectly imprecise (noise).[25]

Our first observation is that, if delaying or advancing disclosure were frictionless in the sense that the scheduling costs were zero (i.e., $c_L = c_E = 0$), there would be no gain in disclosing early—the manager could always wait for the external news before deciding whether to reveal her information. In what follows, we first consider (in Section 6.1) the

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[24] Furthermore, the effect in Subramanyam (1996) would disappear if the prior over firm value, v, were distributed according to an improper prior—so that $v \mid s \sim \mathcal{N}(s, \frac{1}{n})$. By contrast, our result would continue to hold in that setting.

[25] The truth-or-noise structure—employed also by Lewis and Sappington, 1994; Kanodia, Singh, and Spero, 2005; Ottaviani and Sorensen, 2006; Banerjee and Green, 2015; Guttman and Marinovic, 2018; Banerjee, Davis, and Gondhi, 2023; Gonçalves, Libgober, and Willis, 2025—allows for the uncertainty over precision to have the greatest impact possible and reflects several real-world examples: social media tweets and posts as well as mainstream media broadcasts and articles may be substantiated or based on rumors; reports may be truthful or false; announcements may be correct or erroneous.

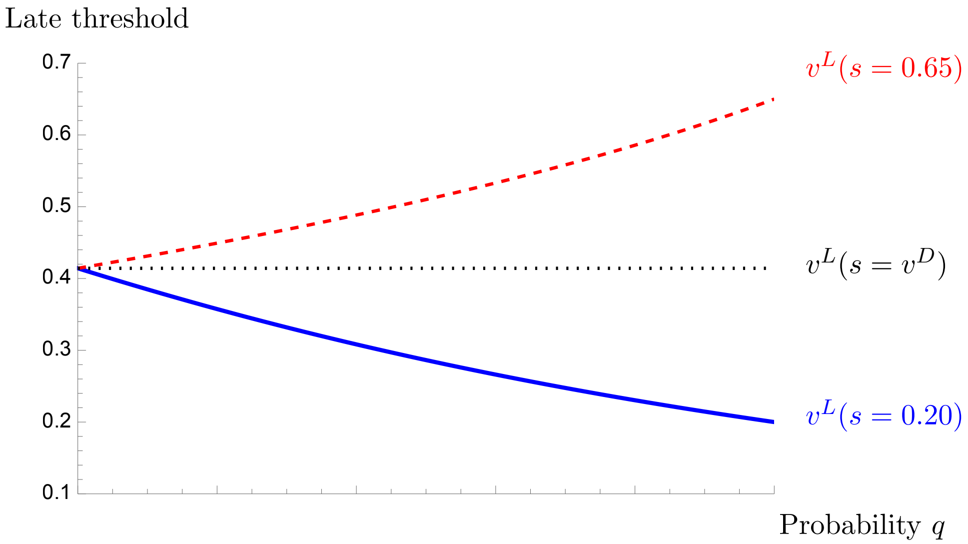

Figure 4: Late disclosure threshold as a function of q

Numerical example where $p = 0.5$ and the signal is either $s = u$ (corresponding to $\phi = \underline{\phi} = 0$) or $s = v$ (corresponding to $\phi = \overline{\phi} = 1$) with $v \sim \mathcal{U}[0,1]$ and $u \sim \mathcal{U}[0,1]$. Here, $v^D = 0.42$.

simple case where the timing choice is cost-free. Then (in Section 6.2), we show that imposing scheduling costs may generate nontrivial decisions at both times.

6.1 Costless Timing Choice and Discussion of Late Disclosure

As previously noted, when choosing the time of disclosure is cost-free, the manager will always delay disclosure.[26] In this scenario, an informed manager responds to the signal (on date 3) by revealing v if the anticipated date-4 disclosure price exceeds the nondisclosure one detailed in (1).

Lemma 3. *Suppose $c_L = c_E = 0$. There exists a unique equilibrium where for any signal realization s the manager discloses on date 3 if the observed value exceeds a threshold $v^L(s) \in V$, and remains silent otherwise.*

Proposition 3.

(i) The late disclosure threshold $v^L(s)$ is increasing in q if $s > v^D$ but decreasing if $s < v^D$;

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[26] Late disclosure can also arise exogenously when macroeconomic news and announcements, mainstream media articles and broadcasts, or social media posts appear unexpectedly (say, before a scheduled conference call). Alternatively, the manager may observe the firm value only after the arrival of external news—the results in Section 6 hold qualitatively under such an alternative timeline.

(ii) If $s \lesseqgtr v^D$ *then* $v^L(s) \gtreqless s$ *and* $v^L(s) \lesseqgtr v^D$*;*

(iii) The late disclosure threshold $v^L(s)$ *has a kink at* $s = v^D$ *such that* $1 > \frac{\partial}{\partial s} v^L(s|s \leq v^D) > \frac{\partial}{\partial s} v^L(s|s > v^D) > 0$.

To understand part (i) of Proposition 3, note that in the face of manager silence, the market expectation of firm value is a convex combination of the price when s is certainly precise, $\lim_{q \to 1} P(s, \varnothing) = s$, and when it is certainly imprecise, $\lim_{q \to 0} P(s, \varnothing) = P(\varnothing) = v^D$. This implies that, when $s = v^D$, the nondisclosure price (and thus the late disclosure threshold) is *independent* of the signal's perceived precision, since it equals v^D no matter what the market conjectures about the manager's disclosure strategy. It turns out that $s = v^D$ is the only signal with this property. For any $s \neq v^D$, the effect of q is nontrivial, as illustrated in Figure 4. In particular, if the signal is sufficiently good ($s > v^D$), higher q implies that investors believe the signal is more likely precise and put more weight on this signal, which increases the nondisclosure price and strengthens the manager's incentives for silence, i.e., $v^L(s)$ increases. The opposite is true when the news is bad ($s < v^D$). Then, higher q means investors put more weight on this bad news, which reduces the nondisclosure price and induces the manager to disclose, i.e., $v^L(s)$ decreases. Put differently, good (bad) external news that is more likely precise discourages (encourages) disclosure.

It seems intuitive that managers respond and try to correct bad external news. Part (ii) of Proposition 3, however, shows that this intuition is not always true. If $s < v^D$, it holds that $v^L(s) > s$; i.e., the manager withholds values that are *better* than those revealed by the external news. Conversely, if $s > v^D$, it holds that $v^L(s) < s$; i.e., the manager discloses values that are *worse* than those revealed by the external news. This result, illustrated in Figure 5, may at first seem perplexing: why would anyone choose not to correct bad news? The answer is simple: the manager discloses when the observed value exceeds the nondisclosure price.[27] Because investors weigh external news with a grain of salt, the latter

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[27] The result that investors take external news with a grain of salt, leading managers to disclose information when investors' expectations fall short of the firm's actual value and remain silent when these expectations

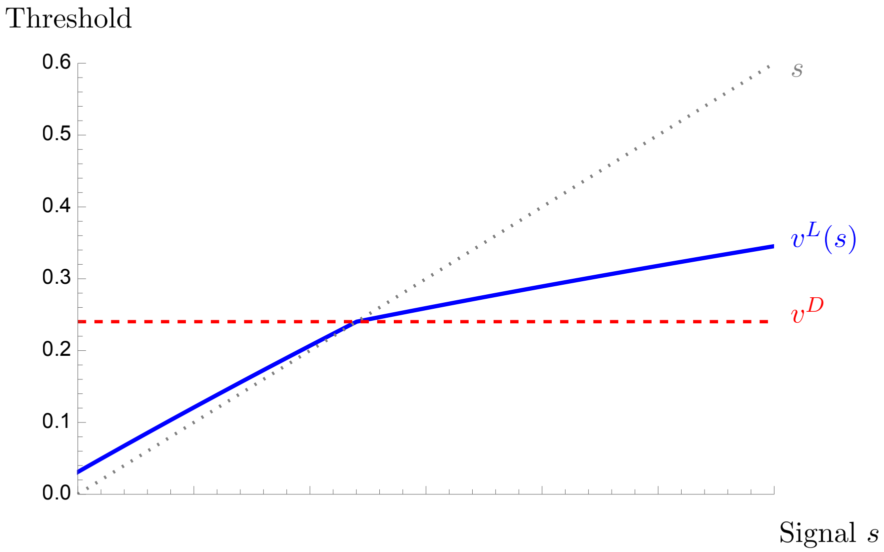

Figure 5: Late disclosure threshold as a function of s

Numerical example where $q = 0.6$, $p = 0.5$, and the signal is either $s = u$ (corresponding to $\phi = \underline{\phi} = 0$) or $s = v$ (corresponding to $\phi = \overline{\phi} = 1$) with $v \sim \mathcal{U}[0, 1]$ and $u \sim \mathcal{U}[0, 1]$. Here, $v^D = 0.42$.

is not identical to s (in particular, investors are skeptical about the news' precision and so underreact to it), remaining silent in the face of some bad news and speaking up to correct downwards some good news may be beneficial.

Lastly, part (iii) of Proposition 3 shows that $v^L(s)$ is more sensitive to bad external news than to good news (see Figure 5). This is driven by the investors' skepticism about good news that are discounted in price formation.[28] Specifically, since better news is perceived as less likely to be precise, the expected value conditional on nondisclosure is less sensitive to changes in the news. As a result, the manager's willingness to disclose—as reflected in $v^L(s)$—responds less to changes in s.

The preceding discussion implies that the nondisclosure price—which in equilibrium equals the disclosure threshold—is more sensitive to bad news. However, this price monotonically increases when the timing choice is frictionless. As we will see in the next section, nonmonotonicity can re-emerge when the choice of timing imposes scheduling costs.

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exceed the actual value, relates to prior work. For example, Zhou (2021) investigates the persistence in disclosure decisions and shows that, when investors' expectations for profitability are lower than the actual profitability, managers disclose to increase investors' expectations. Because investors only gradually adjust their expectations and the underlying profitability remains uncertain, managers' disclosures tend to persist. In our model, a different force operates: the manager's behavior informs investors as to whether their beliefs are based on a more or less precise signal.

[28] The equilibrium threshold is continuous in s, despite this change in sensitivity.

Proposition 3 showed that, relative to the benchmark without external news, low s encourages disclosure, whereas high s discourages it. This asymmetry implies that the overall effect of external news on the probability of disclosure is nontrivial. Assessing this effect requires considering all possible signal realizations and comparing two scenarios: one with the external signal and one without it. In the appendix, we show that:

$$\begin{aligned}&\Pr(d = v|\text{no external news}) - \Pr(d = v|\text{external news})\\ \propto\ &\int_{v_{min}}^{v_{max}} G(v^L(s))g(s)ds - \int_{v_{min}}^{v_{max}} G(v^D)g(s)ds.\end{aligned}$$

Since the sign of this expression is ambiguous, the following result identifies sufficient conditions on the primitives that give rise to either crowding-in or crowding-out.

Proposition 4.

(i) External news crowds out firms' late disclosure when q, the probability that the signal is precise, is sufficiently large, and in addition:

$$\int_{v_{min}}^{v_{max}} G(s)g(s)ds > \int_{v_{min}}^{v_{max}} G(\mu)g(s)ds. \tag{3}$$

(ii) External news crowds in firms' late disclosure when q is sufficiently large, p sufficiently small, and in addition:

$$\int_{v_{min}}^{v_{max}} G(s)g(s)ds < \int_{v_{min}}^{v_{max}} G(\mu)g(s)ds. \tag{4}$$

The overall effect of external news depends on the distribution of firm values.[29] To illustrate, consider the case where the probability density function is given by $g(x) = 2x$ for $x = s, v$. In this setting, values of x are more likely to lie above the mean, and inequality (3) holds. Conversely, if $g(x) = 2(1 - x)$, then x is more likely to fall below the mean, and

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[29] The conditions on p and q in Proposition 4 allow us to derive implications in terms of model primitives rather than endogenous constructs.

inequality (4) holds. More broadly, we predict that external news with uncertain precision tends to crowd out (crowd in) late disclosures when firms are more likely to have high (low) underlying values. Intuitively, managers benefit from silence when external news about their firms is more likely to be good, as it can boost market prices. Managers benefit from disclosure when news is more likely to be bad.

6.2 Costly Timing Choice

We now proceed with the case where at least one of the scheduling costs, c_E or c_L, is strictly positive. Throughout this section, we assume that these costs are not prohibitively high and suppose that the equilibrium disclosure continues to follow a threshold rule.[30] Within the context of our framework, this means that the manager discloses before the signal realization if $v > v^E_{c_E,c_L}$ and discloses after the signal realization if $v^L(s) < v \leq v^E_{c_E,c_L}$.[31] Our focus is on the extent to which price nonmonotonicity is maintained.

We begin by noting that the manager prefers to delay the disclosure decision until after the signal realization if advancing disclosure is costlier, i.e., $c_L < c_E$.

Lemma 4. *Early disclosure never occurs in equilibrium if $c_L < c_E$. The late disclosure decision in this case is as outlined in Section 6.1.*

What are the implications when $c_L > c_E$? In this case, disclosing earlier can allow the manager to reduce the associated scheduling costs. Our next result shows that the availability of the option to delay reduces the likelihood of early disclosure.[32]

Lemma 5. *Suppose that $c_L > c_E$. Then, for any equilibrium $v^E_{c_E,c_L}$, we have $v^E \leq v^E_{c_E,c_L}$, with v^E denoting the equilibrium disclosure threshold outlined in Lemma 2.*

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[30] We establish that such an equilibrium exists for certain cost parameters in the proof of Proposition 5. While threshold equilibria exist more generally than this proposition requires, providing a complete existence result is beyond the scope of our extension.

[31] This includes the case where there is no early disclosure (i.e., if parameters are such that $v^E_{c_E,c_L} \geq v_{max}$) as well as the case where there is no late disclosure (i.e., if $v^L(s) \notin [v_{min}, v^E_{c_E,c_L}]$).

[32] The reason is twofold. First, the presence of a cost ($c_E > 0$) raises the disclosure threshold relative to the one in Section 4 (see Verrecchia, 1983 for a related result involving fixed disclosure costs). Second, the mere availability of a delay option weakens incentives to disclose early—even when $c_E = 0$.

This observation has implications for price nonmonotonicity. To see why, recall that Lemmas 2 and 5 established that $v^E_{c_E,c_L} \geq v^E > v^D$. As a result, increased skepticism about the precision of s in the neighborhood of $v^E_{c_E,c_L}$ should lead to a drop in the market price, following the same logic presented in Section 4. Whether price nonmonotonicity arises ultimately depends on the market's expectations regarding managers with firm values just below $v^E_{c_E,c_L}$. If these managers are expected to disclose late (as is the case when $c_E = c_L = 0$), then the market's beliefs remain constant as s increases from below to above $v^E_{c_E,c_L}$, resulting in a monotonic price function (see the discussion following Proposition 3). However, if c_L is sufficiently large, managers with v just below $v^E_{c_E,c_L}$ prefer to remain silent when $s = v$, choosing to disclose only when s is particularly bad. This shift in disclosure behavior leads to a drop in price as s reaches the threshold, giving rise to the following result:

Proposition 5. *There exists a set of values for c_E and c_L such that*

(a) There is positive probability both that the manager discloses early and that the manager discloses late, and

(b) The price is nonmonotonic around $v^E_{c_E,c_L}$ whenever the manager remains silent in both periods.

For the cost values identified in the proof of Proposition 5, disclosures can occur in either period, leading to the re-emergence of price nonmonotonicity around the early threshold. In particular, as illustrated in Figure 6, the market price of firms remaining silent in both periods exhibits three distinct segments. (i) For any s after which (late) disclosure occurs with positive probability—in the figure, signals below s^L, where s^L is the lowest signal for which late disclosure occurs with zero probability—the market price increases monotonically and is relatively more sensitive to external news.[33] (ii) For $s \in (s^L, v^E_{c_E,c_L})$, the price continues to rise monotonically but with reduced sensitivity. This is depicted by the flatter slope of the price function in this region. (The added gray dotted line has been added to facilitate comparison

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[33] Recall that the late disclosure threshold (unlike early) is a function of s, hence the need to define s^L.

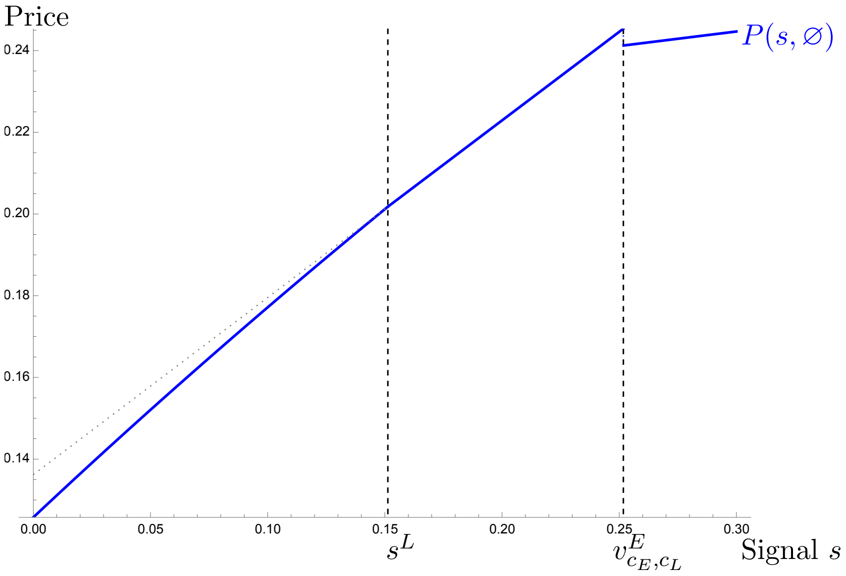

Figure 6: Equilibrium price function under the conditions of Proposition 5

Numerical example where $p = 0.9$, $q = 0.2$, $c_E = 0, c_L = 1/20$, and the signal is either $s = u$ (corresponding to $\phi = \underline{\phi} = 0$) or $s = v$ (corresponding to $\phi = \overline{\phi} = 1$) with $v, s \sim \mathcal{U}[0, 1]$.

of the slopes.) Around the early-disclosure threshold $v^E_{c_E,c_L}$, price nonmonotonicity emerges, characterized by a (discrete due to the truth-or-noise signal structure) drop. (iii) For $s > v^E_{c_E,c_L}$, the price resumes its monotonic increase, though with significantly lower sensitivity to external news.

Under a uniform distribution, we can demonstrate that nonmonotonicity becomes *more pronounced* relative to the case without the option to delay (Sections 4 and 5). This effect is driven by an increase in the early disclosure threshold. Since this upward shift in the threshold holds more generally, we conjecture that the amplification of nonmonotonicity extends to a broader class of distributions. However, since shifts in the disclosure threshold *also* influence the relative probability that s is noise, conditional on s, establishing this result analytically remains intractable.

7 Empirical Predictions

Our model yields several key empirical implications for researchers investigating corporate discretionary disclosure.

First, we find that an increase in the positivity of external news may paradoxically lead to a lower market valuation in the neighborhood of the (early) disclosure threshold, i.e., around the firm values that the market expects an informed manager to have disclosed before the news, which is typically in the intermediate region (Proposition 1 and Proposition 5). As we explained in Section 4, the intuition is rooted in the updating of investors' beliefs and the firm's disclosure behavior: Because the manager discloses only favorable private information, her silence leads investors to take good external news with a grain of salt. Particularly, signals just above the disclosure threshold are perceived to be less precise than those that are just below the threshold. The intuition is that, if external news is precise and good, the manager discloses. Since the manager remains silent, the good news is unlikely to be precise. The described endogenous updating of beliefs is the cause for the sharp decline of market prices in that region. When the external news is either very good or very bad (i.e., away from the disclosure threshold), the market reacts as one would expect: better news lead to higher prices. Summarizing these observations and taking into account when early disclosure occurs in practice, we can formulate our first prediction:

Prediction 1. *The association between market prices of firms that remain silent early on (e.g., when delaying disclosure is relatively costly or before entering a quiet period in the lead-up to an IPO or a business quarter end) and external news with uncertain precision is:*

(i) (weakly) negative if the external news is in the intermediate region;

(ii) positive when the external news is very bad or very good.

Part (ii) of our prediction is relatively straightforward to test empirically. Testing part (i), however, is less so because it requires identifying the precise inflection point where prices deviate from monotonic behavior. One potential avenue could be to construct a measure of the disclosure threshold based on the values disclosed by peers with similar characteristics. Another possibility is to construct such a measure based on historical patterns of the same firm. We appreciate that this is a challenging task but hope that future research could explore the conditions under which nonmonotonic price responses occur and their implications for investor behavior and firm disclosure strategies.[34]

Overall, our first prediction helps explain the mixed empirical findings about stock price reactions to external information. We anticipate that, in cases where the information is released by trusted sources, stock prices of firms whose managers remain silent will generally increase monotonically with news positivity. However, when there is uncertainty about the precision of the source, better external news could lead to a lower price if the firm is silent. This dynamic is increasingly relevant today, when the gradual decline of trusted information providers shapes the way modern firms communicate with their investors.

Second, our model can illuminate market price reactions to external news. It is easy to see from equation (1) that, apart from a knife-edge case, the nondisclosure price differs from the external signal, $P(s, \varnothing) \neq s$. This observation is consistent with Zhang (2006a) and Zhang (2006b), who document that investors underreact to uncertain external news. More importantly, for highly uncertain precision (sufficiently large Δ_ϕ), Proposition 2 establishes that $\frac{\partial P(s,\varnothing|s<s')}{\partial s} > \frac{\partial P(s,\varnothing|s>s'')}{\partial s}$, where $s' < v^E < s''$. This result simply means that investors are more sensitive to bad news (than to good news), due to their perception of it as more likely to be precise. The impetus for this observation is information uncertainty. Thus, the reaction asymmetry disappears when $q = \{0, 1\}$ and is more pronounced when the uncertainty is high. Notably, our results arise for both early and late disclosure settings, so our prediction below does not distinguish between these cases:

Prediction 2. *Market prices of nondisclosing firms are more sensitive to bad (than to good) external news with highly uncertain precision. The magnitude of the reaction asymmetry is positively correlated with the uncertainty about news precision.*

The existence of asymmetric price response to external news has been empirically documented in diverse settings (Aggarwal and Schirm 1998; Goldberg 2013; Blot, Hubert, and

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[34] Without identification of disclosure threshold, we expect the regression coefficients of intermediate external news on market prices of nondisclosing firms to be insignificant due to the forces identified in our study.

Labondance 2024; Capkun, Lou, Otto, and Wang 2023; Xu and You 2025). However, these studies do not address whether the observed phenomenon emerges due to the economic forces in our model—specifically, the nuanced updating of investor beliefs. Future research could incorporate observable proxies for uncertainty in precision. A particularly suitable setting for such an analysis is corporate rumors, given their highly uncertain precision. To our knowledge, no studies in the burgeoning rumors literature (e.g., Alperovych, Cumming, Czellar, and Groh 2021; Cai, Quan, and Zhu 2023; Liu and Moss 2025) have assessed the variation in the price response coefficients in the rumors' content. This could be a promising avenue for future empirical research.[35]

Third, our analysis sheds light on the relationship between the probability of disclosure and external news. Our predictions differ based on whether firms are disclosing early or late. We first recall that the probability of disclosure is given by $p(1 - F(\widehat{v}))$ for $\widehat{v} = \{v^E, v^L(s)\}$. Hence, the effects on the probability of disclosure depend on the disclosure thresholds. In the main part of the analysis (focusing on early disclosure), the mere presence of external news with uncertain precision increases the *early* disclosure threshold, $v^E > v^D$ (Lemma 2). By contrast, the presence of external news may increase or decrease the *late* disclosure threshold, $v^L(s) \lesseqgtr v^D$ if $s \lesseqgtr v^D$ (Proposition 3). The overall effect—before knowing the news—can go in either direction (Proposition 4), but we note that the presence of external news crowds out (crowds in) corporate disclosure when firm values are more likely to be high (low). In addition, we consider how changes in q affect the disclosure thresholds: For *early* disclosure, $\frac{\partial v^E}{\partial q} > 0$. However, when it comes to *late* disclosure, Proposition 3 shows that $\frac{\partial v^L(s)}{\partial q} \lesseqgtr 0$ if $s \lesseqgtr v^D$. Combining our observations, we can formulate the following prediction:

Prediction 3. *The presence of external news with uncertain precision has a nontrivial effect on the probability of corporate disclosure.*

(i) It reduces the probability of disclosure by firms before entering a quiet period in the

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[35] For accounting-related setting, one could also examine the price response coefficient to mandatory releases of peer firm information, where there might be uncertainty as to whether the peer information is relevant for the analyzed firm.

lead-up to an IPO or a business quarter end;

(ii) It reduces (increases) the probability of disclosure by firms responding to already released external news if the external news is good (bad).

(iii) Overall, firms are more (less) likely to respond to external news in industries where firm values are more likely to be low (high).

These effects are exacerbated when the external news is more likely precise.

Consistent with our prediction in part (i), Breuer, Hombach, and Müller (2022) show that increased mandatory disclosures by peer firms can crowd out voluntary disclosures. In line with part (ii), Capkun, Lou, Otto, and Wang (2023) document that the likelihood of firms responding to peer announcements about medical trial outcomes depends on the positivity of the trial outcomes. Future research could explore how variations in the informational environment (driven by regulatory changes or other factors affecting source precision) influence firms' disclosure strategies.

Fourth, Proposition 3 shows, in relation to late disclosures, that $v^L(s) \underset{<}{\geq} s$ if $s \underset{>}{\leq} v^D$ (see also Figure 5), implying that firms may withhold more favorable information than that reflected in the external signal but, at the same time, respond by revealing unfavorable information. This result helps us formulate our next prediction:

Prediction 4. *Firms respond to already released bad (good) external news by withholding (disclosing) some information that is more (less) favorable than the one revealed in the external news.*

Our model suggests that firms may respond to external information with disclosures that could lead to lower stock prices. Although this outcome might at first seem to contradict disclosure theories, we believe such reactive disclosures can occur within a rational framework. For example, Sletten (2012) finds that, following a peer restatement, managers disclose information that results in a price decline. Sletten (2012) notes the absence of a theoretical basis for these disclosures of unfavorable news after a peer's restatement. Our findings provide a potential explanation, suggesting that these disclosures might be a strategic reaction to external news.

Fifth, our analysis of the firm's delayed disclosure (Section 6) has important implications for market price volatility. When external news is bad, managers are more likely to try to correct the market's inference by disclosing, leading to larger shifts in beliefs and prices. By contrast, following good external news, fewer managers disclose, resulting in smaller price movements. Consequently, we anticipate that price volatility following external news is negatively correlated with the positivity of that news. Furthermore, Proposition 3 establishes that the disclosure threshold increases with news precision when signals are good and decreases when theyare bad. As a result, when external news is more likely to be precise—such as when it originates from peer firms or official governmental sources rather than social media—the disparity in price volatility between good and bad news should be even more pronounced.

Prediction 5. *Price volatility is high when the external news is bad and low when the news is good. The difference in volatility is larger for more precise news.*

While empirical research has explored how managers respond to external news (e.g., Sletten 2012; Capkun, Lou, Otto, and Wang 2023), we are not aware of studies that examine the link between external news and subsequent price volatility. Future work could draw on these same settings–adding proxies for external news positivity and precision–to provide the first direct evidence on the nature and extent of their effect on price volatility.

Lastly, our analysis suggests that firms may benefit from delaying their disclosures until after the release of external news, as long as there are no costs associated with delaying. One potential setting to explore this prediction is the one studied by Dambra, Velikov, and Weber (2024), who analyze how the content of monetary announcements influences voluntary disclosures among firms with varying sensitivity to monetary policies. To extend their analysis, researchers might examine whether firms more sensitive to monetary policies are disproportionately more inclined to disclose information after Federal Open Market Committee meetings compared to those less affected by such information. This line of inquiry could yield insights into the strategic timing of disclosures.

Overall, our analysis and predictions can be applied to many settings where the precision of nonstrategic third-party news is questionable. One prominent example is social media, where it is often unclear whether posts are precise or "fake, out of context and straight propaganda" (Kelly 2023). Another example is ESG ratings, the sources for which can be flawed due to different weights, scopes, and measurements (Berg, Kölbel, and Rigobon 2022). Consequently, investors and other stakeholders skeptically interpret this third-party information. This skepticism stems from the rational yet nuanced updating of beliefs by investors, and our model offers a novel rationale to explain this phenomenon.

8 Conclusion

We examine how uncertainty about the precision of external news shapes investor beliefs, managerial disclosure, and market prices. Our findings have implications for investors and firms operating in environments where the credibility of external news—such as traditional media broadcasts, social media posts, and ESG ratings—is uncertain. These findings provide insights into recent empirical work in disclosure (e.g., Sletten 2012; Capkun, Lou, Otto, and Wang 2023) and suggest directions for future research.

Specifically, we find that investors perceive good external news as less precise, reinforcing their belief that managers are withholding unfavorable information. As a result, the stock prices of nondisclosing firms may exhibit a nonmonotonic response, with better news sometimes leading to price declines. Market reactions are also asymmetric, as stock prices respond more strongly to bad news than to good news under uncertainty.

Furthermore, external news influences firms' disclosure strategies in distinct ways. Managers are less likely to disclose before a quiet period but more (less) likely to do so in response to bad (good) news, with this effect intensifying with the credibility of the news. Firms may also strategically withhold disclosures when external news is bad while selectively releasing disclosures that are slightly worse than good external news. Additionally, firms appear more likely to disclose in industries where firm values are lower, suggesting that industry-level dynamics help shape disclosure decisions. Finally, we show that market price volatility rises following bad external news, particularly when the news is more precise.

Together, our findings underscore the importance of considering firm disclosure strategies in settings where the credibility of third-party sources of information varies. Overall, our research highlights the complexities of corporate disclosure in an era where external news is abundant but its precision unclear.

Appendix

Proof of Lemma 1: The first part follows immediately from Milgrom (1981), since strict MLRP implies that $H(v|s,\phi)$ is strictly FOSD-increasing in s, which in particular implies that the posterior expectation of v given s is strictly increasing in s.

For the second part, let the observed signal be $s = s' \in V$. For clarity as s is a continuous variable, we use the limit of measurable intervals and express the joint belief $\Pr(\phi,\kappa|s = s',\varnothing) = \lim_{\delta\to 0}\frac{\Pr(s\in[s',s'+\delta],\varnothing,\phi,\kappa)}{\Pr(s\in[s',s'+\delta],\varnothing)}$ for $\phi \in \{\underline{\phi},\overline{\phi}\}$, $\kappa \in \{I, U\}$, $\delta > 0$, and then consider $\delta \to 0$. Let $A' \equiv [s', s'+\delta]$ and focus on the numerator. We note that, for all ϕ:

$$\begin{aligned}\Pr(s\in A',\varnothing,\phi,I) &= p\cdot\Pr(\phi)\cdot\int_{s\in A'}\int_{v_{min}}^{\widehat{v}} f(s|v,\phi)g(v)dvds;\\ \Pr(s\in A',\varnothing,\phi,U) &= (1-p)\cdot\Pr(\phi)\cdot\int_{s\in A'}\int_{v_{min}}^{v_{max}} f(s|v,\phi)g(v)dvds.\end{aligned}$$

Summing these and using linearity of the integral, we have:

$$\Pr(s\in A',\varnothing,\phi) = \int_{s\in A'}\Pr(\phi)\cdot\left(p\int_{v_{min}}^{\widehat{v}} f(s|v,\phi)g(v)dv + (1-p)\int_{v_{min}}^{v_{max}} f(s|v,\phi)g(v)dv\right)ds.$$

Summing over ϕ, in turn, yields $\Pr(s\in A',\varnothing)$, the denominator in the conditional expectation. We have:

$$\Pr(\phi,I|s\in A',\varnothing) = \frac{p\cdot\Pr(\phi)\cdot\int_{s\in A'}\int_{v_{min}}^{\widehat{v}} f(s|v,\phi)g(v)dvds}{\sum_{\widetilde{\phi}\in\{\underline{\phi},\overline{\phi}\}}\int_{s\in A'}\Pr(\widetilde{\phi})\cdot\left(p\int_{v_{min}}^{\widehat{v}} f(s|v,\widetilde{\phi})g(v)dv + (1-p)\int_{v_{min}}^{v_{max}} f(s|v,\widetilde{\phi})g(v)dv\right)ds}.$$

Note that $\int_{s\in A'}\int_{v_{min}}^{v_{max}} f(s|v,\widetilde{\phi})g(v)dv$ denotes the probability that $s\in A'$ when the correlation parameter is $\widetilde{\phi}$. Let $H(v|s,\phi)$ denote the cumulative distribution over v conditional on s. By Bayes' Rule, the density for this distribution is:

$$h(v|s,\phi) = \frac{f(s|v,\phi)g(v)}{\int_{v_{min}}^{v_{max}} f(s|\widetilde{v},\phi)g(\widetilde{v})d\widetilde{v}}.$$

Then, $H(\widehat{v}|s\in A',\phi) = \frac{\int_{s\in A'}\int_{v_{min}}^{\widehat{v}} f(s|v,\phi)g(v)dv}{\int_{s\in A'}\int_{v_{min}}^{v_{max}} f(s|v,\phi)g(v)dv}$, since the numerator is the probability that $s\in A'$ and $v\le\widehat{v}$, and the denominator is the probability that $s\in A'$ (both given correlation parameter $\widetilde{\phi}$). Dividing each term in the sum in the denominator by $\int_{s\in A'}\int_{v_{min}}^{v_{max}} f(s|v,\widetilde{\phi})g(v)dv = \Pr(s\in A' \mid \widetilde{\phi})$, and similarly for the numerator, we obtain:

$$\Pr(\phi, I|s \in A', \varnothing) = \frac{p \cdot \Pr(\phi)\Pr(s \in A'|\phi) \cdot H(\widehat{v}|s \in A', \phi)}{\sum_{\widetilde{\phi} \in \{\underline{\phi}, \overline{\phi}\}} \Pr(\widetilde{\phi})\Pr(s \in A'|\widetilde{\phi}) \cdot \Big(pH(\widehat{v}|s \in A', \widetilde{\phi}) + (1-p)\Big)}.$$

Note that we can write $\Pr(\widetilde{\phi})\Pr(s \in A'|\widetilde{\phi}) = \Pr(\widetilde{\phi}|s \in A')\Pr(s \in A')$ for each $\widetilde{\phi}$. Making this substitution in both the numerator and the denominator, we can cancel $\Pr(s \in A')$ and obtain:

$$\Pr(\phi, I|s \in A', \varnothing) = \frac{p \cdot \Pr(\phi|s \in A') \cdot H(\widehat{v}|s \in A', \phi)}{\sum_{\widetilde{\phi} \in \{\underline{\phi}, \overline{\phi}\}} \Pr(\widetilde{\phi}|s \in A') \cdot \Big(pH(\widehat{v}|s \in A', \widetilde{\phi}) + (1-p)\Big)}.$$

We now take $\delta \to 0$, and note that we can approximate each integral by the value of the integrand multiplied by δ; further note that for all ϕ, $\lim_{\delta \to 0} H(\widehat{v}|s \in A', \phi) \to H(\widehat{v}|s', \phi)$. We therefore obtain:

$$\Pr(\phi, I|s = s', \varnothing) = \frac{p \cdot \Pr(\phi|s') \cdot H(\widehat{v}|s', \phi)}{\sum_{\widetilde{\phi}} \Pr(\widetilde{\phi}|s') \cdot (pH(\widehat{v}|s', \phi) + (1-p))}.$$

Following identical steps, we similarly obtain:

$$\Pr(\phi, U|s = s', \varnothing) = \frac{(1-p) \cdot \Pr(\phi|s')}{\sum_{\widetilde{\phi}} \Pr(\widetilde{\phi}|s') \cdot (pH(\widehat{v}|s', \phi) + (1-p))}.$$

Since our model assumes that v given s converges in distribution to (a) an independent draw according to G as $\phi \to 0$ and (b) a point mass on s as $\phi \to 1$, we have:

$$\lim_{\phi \to 0} H(\widehat{v}|s', \phi) = G(\widehat{v}), \qquad \lim_{\phi \to 1} H(\widehat{v}|s', \phi) = 1.$$

Furthermore, $Pr(\overline{\phi}|s'), 1 - Pr(\underline{\phi}|s') \to q$ as $\Delta_\phi \to 1$. Using this observation and our previous calculations, we compute the beliefs in the limit as $\Delta_\phi \to 1$:

$$\begin{aligned}
\Pr(U, \overline{\phi} \to 1|s = s', \varnothing) &= \frac{(1-p)q}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)}; \\
\Pr(U, \underline{\phi} \to 0|s = s', \varnothing) &= \frac{(1-p)(1-q)}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)}; \\
\Pr(I, \overline{\phi} \to 1|s = s', \varnothing) &= \frac{pq}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)}; \\
\Pr(I, \underline{\phi} \to 0|s = s', \varnothing) &= \frac{pG(\widehat{v})(1-q)}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)}.
\end{aligned}$$

Fixing any ε sufficiently small, we let $\bar{\Delta}(\varepsilon, s')$ be such that each of these four quantities are within ε of the limit whenever $\Delta_\phi > \bar{\Delta}(\varepsilon, s')$, which holds since each of these converge.

For s'', the limit is almost the same except that $\lim_{\phi \to 1} H(\widehat{v}|s'', \phi) = 0$ (instead of 1). Making the same substitutions, we have:

$$\begin{aligned}
\Pr(U, \overline{\phi} \to 1 | s = s'', \varnothing) &= \frac{(1-p)q}{q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)}; \\
\Pr(U, \underline{\phi} \to 0 | s = s'', \varnothing) &= \frac{(1-p)(1-q)}{q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)}; \\
\Pr(I, \overline{\phi} \to 1 | s = s'', \varnothing) &= 0; \\
\Pr(I, \underline{\phi} \to 0 | s = s'', \varnothing) &= \frac{pG(\widehat{v})(1-q)}{q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)}.
\end{aligned}$$

We can similarly define $\bar{\Delta}(\varepsilon, s'')$ to be such that each of these four quantities are within ε of the limit whenever $\Delta_\phi > \bar{\Delta}(\varepsilon, s')$, which holds since each of these converge.

Note that all three quantities other than $\Pr(I, \overline{\phi} \to 1 | s, \varnothing)$ are larger for $s = s''$ than $s = s'$, since the denominator for each fraction is smaller; for $\Pr(I, \overline{\phi} \to 1 | s, \varnothing)$ the conclusion is reversed, since this is 0 for $s = s''$ and positive for $s = s'$. Thus, take any ε such that:

$$\varepsilon < \frac{|\Pr(\kappa, \phi | s'', \varnothing) - \Pr(\kappa, \phi | s', \varnothing)|}{2}$$

Setting $\bar{\Delta} = \max\{\bar{\Delta}(\varepsilon, s'), \bar{\Delta}(\varepsilon, s'')\}$, all claimed inequalities hold for any $\Delta > \bar{\Delta}$, proving the second part of the lemma.

Proof of Lemma 2: *Part (i)*: As our proof of Part (i) is quite involved, we first present an outline of the steps we follow:

- In Step One, we show that in any threshold equilibrium, $P(s, \varnothing)$ has a slope less than 1 as a function of s, at any value of s where it is continuous and where the market's belief over the probability disclosure that occurs when $\phi = 1$ is constant.

- Step Two shows that there cannot be any threshold equilibrium where the corresponding equilibrium price would "jump up."

- We then show in Step Three that there exists some $v^E \in (v^D, \mu)$ such that the manager is indifferent between disclosing and not, when the market conjectures that indifferent managers remain silent, assuming $\underline{\phi} = 0$

- Step Four verifies that the same conclusion from Step Three holds whenever $\underline{\phi}$ is sufficiently close to 0.

- Finally, Step Five concludes that the indifference condition does indeed determine a threshold equilibrium; that is, if the manager is indifferent between disclosing at v^E and not, then there will be a strict preference for disclosure when $v > v^E$ and a strict preference for nondisclosure when $v < v^E$.

Note that the first step is needed since the manager's payoff under nondisclosure is nonconstant in v.

Step One: This part follows from examination of the price, given the structure of our problem and our restriction to signals s such that the belief over ϕ is constant. Let θ be the market's conjecture of the probability the manager discloses when $\phi = 1$. We emphasize that θ will also play a role in later steps of the proof. The price is:

$$\frac{q(1-p\theta)s + (1-q(1-p\theta))\left((1-p)\int_{v_{min}}^{v^{max}} vh(v|s,\underline{\phi})dv + p\int_{v_{min}}^{v^E} vh(v|s,\underline{\phi})dv\right)}{q(1-p\theta) + (1-q(1-p\theta))\left((1-p) + p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv\right)},$$

which we rewrite as:

$$\frac{\frac{q(1-p\theta)}{(1-p)+p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv}s + (1-q(1-p\theta))\overbrace{\left(\frac{(1-p)\int_{v_{min}}^{v^{max}} vh(v|s,\underline{\phi})dv + p\int_{v_{min}}^{v^E} vh(v|s,\underline{\phi})dv}{(1-p)+p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv}\right)}^{:=t(s)}}{\frac{q(1-p\theta)}{(1-p)+p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv} + (1-q(1-p\theta))}. \tag{5}$$

Note that $t(s)$ is the expected value of v conditional on nondisclosure *and* $\phi = \underline{\phi}$. Thus, the price can be written as $\alpha(s)s + (1-\alpha(s))t(s)$, whose derivative (which exists, since θ is assumed constant) is $\alpha(s) + (1-\alpha(s))t'(s) + \alpha'(s)(s - t(s))$. We note that the maximum values of $\alpha'(s)$ and $t'(s)$, taken over s, both approach 0 as $\underline{\phi} \to 0$: To see this, we compute

$$\begin{aligned} t'(s) = &\frac{(1-p)\frac{\partial}{\partial s}\int_{v_{min}}^{v^{max}} vh(v|s,\underline{\phi})dv + p\frac{\partial}{\partial s}\int_{v_{min}}^{v^E} vh(v|s,\underline{\phi})dv}{(1-p)+p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv} \\ &- \frac{\left((1-p)\int_{v_{min}}^{v^{max}} vh(v|s,\underline{\phi})dv + p\int_{v_{min}}^{v^E} vh(v|s,\underline{\phi})dv\right)p\frac{\partial}{\partial s}\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv}{((1-p)+p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv)^2}. \end{aligned}$$

In particular, note that the denominators in this sum are bounded away from 0. Furthermore, Assumption 1 implies that $\frac{\partial}{\partial s}\int_{v_{min}}^{v^{max}} vh(v|s,\underline{\phi})dv$, $\frac{\partial}{\partial s}\int_{v_{min}}^{v^E} vh(v|s,\underline{\phi})dv$, and $\frac{\partial}{\partial s}\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv$

all converge to 0 uniformly in s as $\underline{\phi} \to 0$, since v_{min}, v_{max} and v_E are bounded and $\max_{s,v} |\frac{\partial}{\partial s} h(v|s, \underline{\phi})| \to 0$ (together with Leibniz's rule, which allows us to bring the derivative under the integral sign since the endpoints are constant in s.). Since in addition $(1-p)\int_{v_{min}}^{v^{max}} vh(v|s,\underline{\phi})dv + p\int_{v_{min}}^{v^E} vh(v|s,\underline{\phi})dv$ is bounded above by v_{max}, we have that $\max_s t'(s) \to 0$. A similar argument applies to $\alpha(s)$, since the derivative is proportional to a bounded constant times $\frac{\partial}{\partial s}\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv$.

However, for all $p, q \in (0,1)$, there exists some $x > 0$ such that $\alpha(s) < 1 - x$, which follows from inspection of $\alpha(s)$. Furthermore, $s - t(s)$ is bounded since $s, t(s) \in [v_{min}, v_{max}]$. We conclude that there exists Δ_ϕ such that $\alpha(s) + (1 - \alpha(s))t'(s) + \alpha'(s)(s - t(s)) < 1$, whenever $\underline{\phi} < 1 - \Delta_\phi$. In addition, we can take Δ_ϕ such that $t'(s) < 1$ for all s, so that the price function is increasing. Henceforth we assume that this condition is satisfied.

Step Two: We first present the following sublemma:

Lemma A.1. *For any arbitrary v^E such that the manager discloses above v^E (and is silent otherwise), there exists a single signal, which we denote $s^D(v^E)$, such that the expectation of v conditional on s^D and nondisclosure is constant in the belief about precision.*

Proof. When $s = s^D(v^E)$, where $s^D(v^E)$ is as defined in the lemma, the market price should be equal to $s^D(v^E)$. Otherwise, changing the belief about ϕ would affect the market price because the higher the probability that $\phi = 1$ the closer this expectation will be to $s^D(v^E)$. If the market believes that information included in precise signals is never disclosed, then we compute:

$$s^D(v^E) = \frac{q(1-p\theta)s^D(v^E) + (1 - q(1-p\theta))\left((1-p)\int_{v_{min}}^{v^{max}} vh(v|s^D(v^E),\underline{\phi})dv + p\int_{v_{min}}^{v^E} vh(v|s^D(v^E),\underline{\phi})dv\right)}{q(1-p\theta) + (1-q(1-p\theta))\left((1-p) + p\int_{v_{min}}^{v^E} h(v|s^D(v^E),\underline{\phi})dv\right)}. \tag{6}$$

We can then solve for s^D as:

$$s^D(v^E) = \frac{(1-p)\int_{v_{min}}^{v^{max}} vh(v|s^D(v^E),\underline{\phi})dv + p\int_{v_{min}}^{v^E} vh(v|s^D(v^E),\underline{\phi})dv}{(1-p) + p\int_{v_{min}}^{v^E} h(v|s^D(v^E),\underline{\phi})dv}. \tag{7}$$

We note that, varying the market belief that the information of precise signals is disclosed is equivalent to changing the value of q in equation (6). But as we have seen, the value for s^D does not depend on q, and this value is always uniquely defined; to see uniqueness of $s^D(v^E)$, note that the right-hand side is $t(s^D(v^E))$, and that we take $\underline{\phi}$ so that $\max_s t'(s) < 1$. But while the right-hand side has a slope less than or equal to 1, the left-hand side is simply $s^D(v^E)$ which has a slope equal to 1. Thus there is a unique intersection point. □

When $s = s^D(v^E)$, the market price is constant in the conjecture of the probability that managers disclose when $v = s^D$. In particular, note that $s^D(v^E) = t(s^D(v^E))$; since the slope of $t(s)$ is less than 1, we have that if $s < s^D(v^E)$, then $t(s) > s$, and (5) is higher when $\theta = 1$ compared to when $\theta = 0$, since the price is a convex combination of two terms (s and $t(s)$) and more weight is put on the higher one (i.e., $t(s)$—since $s < s^D(v^E)$) when $\theta = 1$.

We now define the value, which we denote by v^*, at which beliefs are constant when s and v^E are both equal to that value. We emphasize that in the special case where $\underline{\phi} = 0 = 1 - \overline{\phi}$, we have $v^* = v^D$, and that this observation is important later in the proof. However, allowing for $\underline{\phi} > 0$ requires a generalization; hence the introduction of v^*. We take the derivative of $m(v^E) := \mathbb{E}[v|(\kappa = U \text{ or } v \leq v^E), \underline{\phi}, s^D(v^E)] - v^E$. We have:

$$\frac{d}{dv^E} m(v^E) = \frac{d}{dv^E} \mathbb{E}[v|(\kappa = U \text{ or } v \leq v^E), \underline{\phi}, s^D(v^E)] + t'(s^D(v^E)) \frac{d}{dv^E} s^D(v^E) - 1;$$

We now argue that $h(v \mid s, \underline{\phi})$ is log-concave. Recall that:

$$h(v \mid \underline{\phi}, s) = \frac{f(s \mid v, \underline{\phi}) g(v)}{\int_{v_{min}}^{v_{max}} f(s \mid \widetilde{v}, \underline{\phi}) g(\widetilde{v}) d\widetilde{v}}.$$

Note that the denominator is a constant in v, and that the numerator is the product of two functions that are log-concave in v and hence is itself log-concave in v. Thus, the distribution of v given s is therefore log-concave.

We now present a second sublemma; we briefly note that this result may be of interest for disclosure models more generally:

Lemma A.2. *Set $q = 0$, so that the external signal is imprecise with certainty. If g is log-concave, then $\mathbb{E}[v|v \leq \overline{v}]$ is increasing in $\overline{v}$ at a rate less than 1.*

Proof. Note that the setting where the external signal is imprecise with certainty is equivalent to the standard setting where there is no external signal at all. Computing the derivative of $\mathbb{E}[v|v \leq \overline{v}]$ with respect to $\overline{v}$, we see that it is

$$\frac{pg(\overline{v})(\overline{v} - \mathbb{E}[v|v \leq \overline{v}])}{(1 - p + pG(\overline{v}))}.$$

If $\overline{v} < v^D$, then this expression is negative, and hence less than 1 for all p such that $\overline{v} < v^D$. Otherwise, for all $\overline{v}$, the derivative of this expression with respect to p is proportional to

$$(1-p+pG(\overline{v})) \left(g(\overline{v})(\overline{v} - \mathbb{E}[v|v \leq \overline{v}]) - pg(v) \frac{\partial}{\partial p} \mathbb{E}[v|v \leq \overline{v}] \right) + pg(\overline{v})(\overline{v} - \mathbb{E}[v|v \leq \overline{v}])(1 - G(\overline{v})).$$

If $\overline{v} > \mathbb{E}[v|v \leq \overline{v}]$, then it is immediate that this expression is positive for all p: this observation that uses that $\mathbb{E}[v|v \leq \overline{v}]$ is decreasing in p (see also Kartik, Lee, and Suen 2021 and references cited therein). Thus, since the slope is increasing in p, it is less than the slope in the case that $p = 1$ (which Bagnoli and Bergstrom 2005 imply is less than 1), we have the slope is less than 1 for all $p \in (0, 1)$, as claimed. $\square$

Lemma A.2 implies $\frac{d}{dv^E}\mathbb{E}[v|(\kappa = U \text{ or } v \leq v^E), \underline{\phi}, s^D(v^E)] < 1$. In fact, since the support of v is compact, this derivative is uniformly bounded away from 1, since first, the derivative is continuous due to the smoothness of f and g, and second, the maximum of a continuous function is achieved on a compact set.[36] Furthermore, $\frac{d}{dv^E}s^D(v^E)$ is bounded. This claim follows from considering the derivative of the right-hand side of (7) when $\underline{\phi} = 0$, which is $\frac{d}{dv^E}\mathbb{E}[v|(\kappa = U \text{ or } v \leq v^E), \underline{\phi}, s^D(v^E)]$ since $t'(s) = 0$ for $\underline{\phi} = 0$. When $\underline{\phi} > 0$, we have $s'_D(v^E) = \frac{\frac{d}{dv^E}\mathbb{E}[v|(\kappa=U \text{ or } v\leq v^E),\underline{\phi},s^D(v^E)]}{1-t'(s)}$, and since we have already taken $\underline{\phi}$ so that $1 - t'(s)$ is uniformly bounded away from 1, this derivative is bounded uniformly.

In particular, since the numerator is less than 1, we can ensure that $s'_D(v^E) < 1$, possibly increasing Δ_ϕ if necessary. Henceforth, we assume that Δ_ϕ is such that this is satisfied, and that $\frac{d}{dv^E}m(v^E) < 0$ for $\underline{\phi} < 1 - \Delta_\phi$; these properties will be useful later.

Now, since we have $m(v_{min}) > 0$ and $m(v_{max}) < 0$, we can define v^* to be the unique value such that $m(v^*) = v^*$. Whenever $v^E < v^*$, we have that $v^E < s^D(v^E)$ since $s^D(v^E)$ has a slope less than 1. In particular, the market price is increasing in θ when $s = v^E < v^*$; we emphasize this conclusion as it is an important part of the argument below.

Toward contradiction, suppose we had some threshold equilibrium with $v^E < v^*$. Note that if the market conjectures that managers with $v < v^E$ do not disclose, then we have, whenever $v < v^E$:

$$qP(v, \varnothing) + (1 - q)\mathbb{E}_{s \sim f(s|v,\underline{\phi})}[P(s, \varnothing)|\underline{\phi}, v] > v,$$

while if managers do disclose when $v > v^E$, then we have:

$$qP(v, \varnothing) + (1 - q)\mathbb{E}_{s \sim f(s|v,\underline{\phi})}[P(s, \varnothing)|\underline{\phi}, v] < v.$$

Taking limits as we approach v^E, we note that the right-hand side of these inequalities both approach v^E. Note that $\mathbb{E}_{s \sim f(s|v,\underline{\phi})}[P(s, \varnothing)|\underline{\phi}, v]$ is continuous in v; in fact, it has a bounded derivative:

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[36] To see the first point, we note that $h(v|s, \phi) = \frac{f(s|v,\phi)g(v)}{\int_{v_{min}}^{v_{max}} f(s|v,\phi)g(v)dv}$, and since $f(s|v, \phi) > 0$ for some v, we have the denominator is never 0. Thus, the derivative is always finite everywhere it is defined, and hence continuous.

$$\left|\frac{d}{dv}\int_{vmin}^{v_{max}} P(s,\varnothing)f(s|v,\underline{\phi})ds\right| \leq \int_{v_{min}}^{v_{max}} |P(s,\varnothing)|\cdot|\frac{d}{ds}f(s|v,\underline{\phi})|ds$$
$$\leq \max\{v_{max},-v_{min}\}\int_{v_{min}}^{v_{max}} |\frac{d}{ds}f(s|v,\underline{\phi})|ds.$$

Now, continuity of the expectation conditional on $\underline{\phi}$ in v implies that $\lim_{v\uparrow v^E}\mathbb{E}_{s\sim f(s|v,\underline{\phi})}[P(s,\varnothing)|\underline{\phi},v] = \lim_{v\downarrow v^E}\mathbb{E}_{s\sim f(s|v,\underline{\phi})}[P(s,\varnothing)|\underline{\phi},v]$. Therefore, the price function must satisfy:

$$\lim_{v\uparrow v^E} P(v,\varnothing) \geq \lim_{v\downarrow v^E} P(v,\varnothing) \tag{8}$$

This inequality is a contradiction; inspecting the price function, we note that $\lim_{v\uparrow v^E} P(v,\varnothing)$ and $\lim_{v\downarrow v^E} P(v,\varnothing)$ differ only in terms of the market's belief that disclosure occurs when $\phi=\overline{\phi}$ (by continuity of the expectations); (8) implies that at v^E the price would be higher if the market conjectured $\theta=0$ instead of $\theta=1$, i.e., the price is decreasing in θ. However, we have argued that when $v^E<v^*$, this is *strictly increasing* in θ. This contradiction establishes that we cannot have $v^E<v^*$.

Step Three: We now focus on the case where $\underline{\phi}=0$. We show that in this case, if the market conjectures a threshold $v^E=v^D$, then a manager with value $v=v^D$ will strictly prefer to not disclose; we also show that if the market conjectures a threshold $v^E=\mu$, then a manager with value $v=\mu$ will strictly prefer to disclose.

Step Three, Part (a): The indifference point must be below μ: We first argue that if we were to have $v^E=\mu$, then a manager with $v=v^E$ would strictly prefer to disclose. To see this, we note:

- The manager's payoff when disclosing approaches μ as the firm's value approaches μ, but
- The expected nondisclosure payoff is strictly less than μ as the disclosure threshold approaches μ.

While the former claim is immediate, to show the latter we analyze the price as a function of the signal:

$$P(s,\varnothing)=\frac{q\cdot s(1-p\mathbb{1}_{s>v^E})+(1-q)((1-p)\mu+p\int_{v_{min}}^{v^E} vg(v)dv)}{q(1-p\mathbb{1}_{s>v^E})+(1-q)(1-p+pG(v^E))}.$$

If the signal is precise and the manager's value is v^E, then $\frac{(1-p)\mu+p\int_{v_{min}}^{v^E} vg(v)dv}{1-p+pG(v^E)} < v^E$ whenever $v^E > v^D$. Thus, at $v^E = \mu > v^D$, the price is less than μ if the signal is precise. On the other hand, when $s \sim G$: if $s > v^E$, then $\frac{(1-p)\mu+p\int_{v_{min}}^{v^E} vg(v)dv}{1-p+pG(v^E)} < v^E < s$. Thus, if the market were to conjecture a lower θ (i.e., a lower probability that the manager disclosed when $\phi = 1$ given the signal), then the price would be even higher. Since for $s \leq v^E$ the market already conjectures that such signals are not disclosed, it follows that for every s,

$$P(s, \varnothing) \leq \frac{q \cdot s + (1-q)((1-p)\mu + p\int_{v_{min}}^{v^E} vg(v)dv)}{q + (1-q)(1-p+pG(v^E))}$$

Taking the expectation (over s) of the right-hand side of this expression yields:

$$\frac{q \cdot \mu + (1-q)((1-p)\mu + p\int_{v_{min}}^{v^E} vg(v)dv)}{q + (1-q)(1-p+pG(v^E))} < \mu, \forall v^E.$$

Thus, if $v^E = \mu$, the price is (bounded above by) a convex combination of terms that are equal to μ and those that are strictly less than μ, as claimed. Thus, whenever the manager's value is μ or greater, the manager will prefer to disclose.

Step Three, Part (b): The indifference point must be above v^D We now argue the disclosure threshold must be strictly above v^D. Slightly abusing notation, let $P^q(s, v = v^E)$ denote the price when the market conjectures threshold v^E, given precision q. We first claim that, at $v^E = v^D$, $P^q(s, v = v^E)$ is strictly convex in q, at any $s \neq v^D$. We also claim that $\frac{d}{dq}\int_{v_{min}}^{v_{max}} P^q(s, v = v^E)g(s)ds$ is equal to 0 at $q = 0$.

Note that $P^q(v^D, v = v^D) = v^D$ by Lemma A.1 and the definition of v^D. Also note that $\int_{v_{min}}^{v_{max}} P^0(s, v = v^D)g(s)ds = v^D$. Taken together, the two above claims imply that $\int_{v_{min}}^{v_{max}} P^q(s, v = v^D)g(s)ds$ is strictly increasing in q, for all $q > 0$. Indeed, for almost every s, $P^q(s, v = v^D)$ is strictly convex in q, so the expectation over s is also strictly convex in q since this expectation preserves convexity. Since the derivative is 0 at $q = 0$, $\int_{v_{min}}^{v_{max}} P^q(s, v = v^D)g(s)ds$ is strictly increasing in q.

So consider the manager's incentives. If the market conjectures a threshold $v^E = v^D$, the manager with firm value at the threshold obtains v^D from disclosing, but:

$$q \cdot P^q(v^D, v = v^D) + (1-q)\int_{v_{min}}^{v_{max}} P^q(s, v = v^D)g(s)ds$$

from not disclosing. However, we have just seen that this is a convex combination of v^D and a term strictly larger than v^D. Thus, the manager strictly gains from not disclosing, showing that $v^E > v^D$.

We now return to the two claims above regarding $P^q(s, v = v^E)$. The claim on strict convexity involves some tedious algebra, which reveals that $\frac{\partial^2}{\partial q^2} P^q(s, v^E) > 0$ if and only if

$$\overbrace{(\mathbb{1}_{s>v^E} - 1 + G(v^E))}^{:=(a)} \overbrace{((1-p)(s-\mu) + p \int_{v_{min}}^{v^E} (s-v)g(v)dv)}^{(b)} > 0.$$

We note that:

$$0 = (1-p)(s-\mu) + p \int_{v_{min}}^{v^E} (s-v)g(v)dv \Leftrightarrow s = \frac{\mu(1-p) + p \int_{v_{min}}^{v^E} vg(v)dv}{1-p+pG(v^E)}.$$

If $v^E = v^D$, then this holds if and only if $s = v^D$. So, (b) is negative if $s < v^D$ and positive if $s > v^D$. Similarly, $\mathbb{1}_{s>v^D} - 1 + G(v^D)$ is positive for $s > v^D$ and negative for $s < v^D$. Putting these two observations together, strict convexity holds.

As for the second claim, we bring the derivative inside the expectation as follows:

$$\begin{aligned} \frac{\partial}{\partial q}\bigg|_{q=0} \int_{v_{min}}^{v_{max}} P^q(s, v = v^E) g(s) ds \propto \int_{v_{min}}^{v_{max}} \bigg(& (1-p+pG(v^E))(s(1-p\mathbb{1}_{s>v^E}) - (1-p)\mu - p \int_{v_{min}}^{v^E} vg(v)dv) \\ & - ((1-p)\mu + p \int_{v_{min}}^{v^E} vg(v)dv)(1 - p\mathbb{1}_{s>v^E} - ((1-p) + pG(v^E)))\bigg) g(s) ds. \end{aligned}$$

The proportionality follows from an application of the quotient rule for derivatives. In particular, the claimed proportionality obtains using the following observations:

- The denominator of $\frac{\partial}{\partial q}\Big|_{q=0} P^q(s, v = v^E)$ is $(1-p+pG(v^E))^2$ (and hence it is independent of s, so that it can be dropped without changing the sign of the derivative).
- The value of the denominator of $P^q(s, v = v^E)$ at $q = 0$ is $1 - p + pG(v^E)$, and the derivative of the denominator with respect to q evaluated at $q = 0$ is $1 - p\mathbb{1}_{s>v^E} - (1 - p + pG(v^E))$.
- The value of the numerator at $q = 0$ is $(1-p)\mu + p \int_{v_{min}}^{v^E} vg(v)dv$, and the derivative of the numerator at $q = 0$ is $s(1 - p\mathbb{1}_{s>v^E}) - (1-p)\mu - p \int_{v_{min}}^{v^E} vg(v)dv$.

To see that $\frac{\partial}{\partial q}\Big|_{q=0} \int_{v_{min}}^{v_{max}} P^q(s, v = v^E) g(s) ds$ is in fact equal to 0, observe that:

$$\int_{v_{min}}^{v_{max}} \left(s(1 - p\mathbb{1}_{s>v^E}) - (1-p)\mu - p \int_{v_{min}}^{v^E} v g(v) dv \right) g(s) ds = 0$$
$$= \int_{v_{min}}^{v_{max}} \left((1 - p\mathbb{1}_{s>v^E} - ((1-p) + pG(v^E))\right) g(s) ds,$$

completing the proof of the second claim. To see these equalities, note that $1 - p\mathbb{1}_{s>v^E} = 1 - p + p\mathbb{1}_{s \leq v^E}$.

Step Four: We now consider the case where $\underline{\phi} \neq 0$. Let ε be the difference between v^D and the expected price when $v^E = v^D$, which the previous step argues is nonzero. If $\overline{\phi} = 1$ and the market conjectures threshold v^E, the market price following signal s is

$$\frac{qs(1 - p\mathbb{1}_{s>v^E}) + (1-q)(p \int_{v_{min}}^{v_{max}} \widetilde{v} h(\widetilde{v}|s, \underline{\phi}) d\widetilde{v} + (1-p) \int_{v_{min}}^{v^E} \widetilde{v} h(\widetilde{v}|s, \underline{\phi}) d\widetilde{v})}{q(1 - p\mathbb{1}_{s>v^E}) + (1-q)(p + (1-p) \int_{v_{min}}^{v^E} h(\widetilde{v}|s, \underline{\phi}))}. \tag{9}$$

Recall that $\max_{s,v} |\frac{\partial}{\partial s} h(v|s, \underline{\phi})| \to 0$ as $\underline{\phi} \to 0$ and that $h(v|s, \underline{\phi}) \to g(v)$ pointwise. In particular, since for all $\underline{\phi}$, $\frac{\partial}{\partial s} h(v|s, \underline{\phi})$ is uniformly bounded, $h(v|s, \underline{\phi})$ is uniformly equicontinuous as a function of s and also uniformly bounded by compactness of the support of s. Therefore, by the Arzela–Ascoli theorem, we have that $h(v|s, \underline{\phi}) \to g(v)$ *uniformly* in s. Thus, given any η, we can choose Δ_ϕ so that $|h(v|s, \underline{\phi}) - g(v)| < \eta$ for all s. In particular, we can pick η so that (9) is within $\varepsilon/2$ of:

$$\frac{q \cdot s(1 - p\mathbb{1}_{s>v^E}) + (1-q)((1-p)\mu + p \int_{v_{min}}^{v^E} \widetilde{v} g(\widetilde{v}) d\widetilde{v})}{q(1 - p\mathbb{1}_{s>v^E}) + (1-q)(1 - p + pG(v^E))},$$

for all s. Thus, the expected price for any $\underline{\phi}$ satisfying this condition will be $\varepsilon/2$ of the expected price when $\underline{\phi} = 0$. For any such $\underline{\phi}$, the manager's expected payoff when disclosing v^D when the market conjectures $v^E = v^D$ will be strictly lower than when not disclosing, by Step Three. An identical argument (possibly requiring increasing Δ_ϕ) shows that the same conclusion can be obtained when the market conjectures threshold $v^E = \mu$.

To conclude, we observe that the expected nondisclosure price for a manager with value $v = v^E$ is continuous in v^E; the existence of a value for which the manager is indifferent between disclosure decisions follows from the intermediate value theorem.

Step Five: We are now ready to show that a threshold equilibrium exists, where the manager with firm value at the threshold is indifferent between disclosure decisions. Note that the manager's payoff from nondisclosure, given a disclosure threshold v^E, is

$$qP(v,\varnothing) + (1-q)\mathbb{E}_{s\sim f(\cdot|\underline{\phi},v)}[P(s,\varnothing)].$$

We first note that in any region where the belief that informed managers disclose precise signals is constant, Step One shows that the slope of $P(v,\varnothing)$ is less than 1 in v; on the other hand, recall that Step Two showed that

$$\frac{d}{dv}\mathbb{E}_{s\sim f(s|v,\underline{\phi})}[P(s,\varnothing)] \leq \max\{v_{max}, -v_{min}\}\int_{v_{min}}^{v_{max}} |\frac{d}{ds}f(s|v,\underline{\phi})|ds.$$

Since $\max_s \int_{v_{min}}^{v_{max}} |\frac{d}{ds}f(s|v,\underline{\phi})|ds \to 0$, we can also ensure that Δ_ϕ is such that the right-hand side of this inequality is less than 1.

So suppose $v < v^E$. Since $v < v^E$, the belief that informed managers disclose precise signals when $s = v$ is indeed constant, and therefore the nondisclosure payoff changes at a rate less than 1, meaning that any such manager would strictly prefer to not disclose (since the disclosure payoff decreases by more than the nondisclosure payoff as v decreases from v^E). On the other hand, if $v > v^E$, then since $v^E > v^*$, $P(v,\varnothing)$ drops discontinuously at v^*, as argued in Step Two. Thus, since the nondisclosure payoff first drops discontinuously at v^E and then increases at a rate less than 1, any manager with $v > v^E$ will strictly prefer to disclose. Therefore, v^E forms an equilibrium.

Part (ii): We show that there can only be one value of v^E such that the manager is indifferent between disclosing and not. We first consider the expected price when the signal is imprecise:

$$\int_{v_{min}}^{v^E} \frac{q\cdot s + (1-q)((1-p)\mu + p\int_{v_{min}}^{v^E} vg(v)dv)}{q + (1-q)(1-p+pG(v^E))} g(s)ds$$
$$+ \int_{v^E}^{v_{max}} \frac{q(1-p)\cdot s + (1-q)((1-p)\mu + p\int_{v_{min}}^{v^E} vg(v)dv)}{q(1-p) + (1-q)(1-p+pG(v^E))} g(s)ds,$$

which we rewrite as:

$$G(v^E)\,\overbrace{\frac{q\cdot\mathbb{E}[v|v\leq v^E]+(1-q)((1-p)\mu+p\int_{v_{min}}^{v^E}vg(v)dv)}{q+(1-q)(1-p+pG(v^E))}}^{:=k_1(v^E)}$$

$$+(1-G(v^E))\,\overbrace{\frac{q(1-p)\cdot\mathbb{E}[v|v\geq v^E]+(1-q)((1-p)\mu+p\int_{v_{min}}^{v^E}vg(v)dv)}{q(1-p)+(1-q)(1-p+pG(v^E))}}^{k_2(v^E)}$$

We claim that $k_1(v^E)$ and $k_2(v^E)$ are both increasing at a rate less than 1. Furthermore, $k_2(v^E) > k_1(v_E)$ for all v^E; indeed, we have $\mathbb{E}[v|v \leq v^E] < \mathbb{E}[v|v \geq v^E]$ for all v^E, and $k_1(v^E)$ is a convex combination of $\mathbb{E}[v|v \leq v^E]$ with $\frac{(1-p)\mu+p\int_{v_{min}}^{v_E}vg(v)dv}{1-p+pG(v^E)}$ and $k_2(v^E)$ is a convex combination of $\mathbb{E}[v|v \geq v^E]$ with $\frac{(1-p)\mu+p\int_{v_{min}}^{v_E}vg(v)dv}{1-p+pG(v^E)}$. Thus the overall derivative is:

$$G(v^E)\,\overbrace{k_1'(v^E)}^{<1}+(1-G(v^E))\,\overbrace{k_2'(v^E)}^{<1}+\overbrace{G'(v^E)}^{+}\,\overbrace{(k_1(v_E)-k_2(v_E))}^{-}<1.$$

Thus it suffices to show the claim that $k_1(v^E)$ and $k_2(v^E)$ increase at a rate less than 1. We start with $k_2(v^E)$. Define $P^{DYE}(v_E) = \frac{(1-p)\mu+p\int_{v_{min}}^{v_E}vg(v)dv}{1-p+pG(v^E)}$ to be the Dye price given threshold v_E. By Lemma A.2, $P^{DYE}(v_E)$ has a slope less than 1; on the other hand, $k_2(v_E)$ is a convex combination of $P^{DYE}(v_E)$ and $\mathbb{E}[v|v \geq v^E] > P^{DYE}(v_E)$ (where the inequality holds since $\mathbb{E}[v|v \leq v^E] < \mu < \mathbb{E}[v|v \geq v^E]$ and $P^{DYE}(v^E)$ is a convex combination of μ and $\mathbb{E}[v|v \leq v^E]$). Since $\mathbb{E}[v|v \geq v^E]$ increases in v^E at a rate less than 1 (by Theorem 6 in Bagnoli and Bergstrom 2005), we have that (i) $k_2(v^E)$ is a convex combination of two functions which both have slope less than 1, and (ii) as v^E increases, more weight is put on the smaller function, $P^{DYE}(v_E)$. These observations imply $k_2(v^E)$ increases at a rate less than 1.

For $k_1(v_E)$, let $\alpha(v^E) = \frac{q}{q+(1-q)(1-p+pG(v_E))}$ and $\beta(v^E) = \frac{(1-q)(1-p)}{q+(1-q)(1-p+pG(v_E))}$. Then we can write:

$$k_1(v^E) = \alpha(v^E)\mathbb{E}[v|v \leq v^E] + \beta(v^E)\mu + (1-\alpha(v^E)-\beta(v^E))\mathbb{E}[v|v \leq v^E].$$

Taking the derivative, we have:

$$\begin{aligned} k_1'(v_E) = \alpha(v^E)\cdot\frac{\partial}{\partial v^E}\mathbb{E}[v|v \leq v^E] + \beta(v^E)\cdot 0 + (1-\alpha(v^E)-\beta(v^E))\cdot\frac{\partial}{\partial v^E}\mathbb{E}[v|v \leq v^E] \\ + \alpha'(v^E)(\mathbb{E}[v|v \leq v^E]) + \beta'(v^E)\mu - (\alpha'(v^E)+\beta'(v^E))\mathbb{E}[v|v \leq v^E] \end{aligned}$$

By log concavity, the first line is the average of terms that are all less than 1, and hence is itself less than 1. The second line reduces to $\beta'(v^E)(\mu - \mathbb{E}[v|v \leq v^E])$. However, $\beta'(v^E) < 0$ and $\mu > \mathbb{E}[v|v \leq v^E]$, so that this line is negative. Hence the derivative is the sum of a term that is less than 1 and a negative term, and thus is less than 1.

To summarize, these claims show that the expected price if the signal is imprecise is increasing in v^E at a rate less than 1.

We now consider the expected price when the signal is precise. To obtain the price in this case, we set $s = v^E$ in the expression for the price function, which yields

$$\frac{qv^E + (1-q)((1-p)\mu + p\int_{v_{min}}^{v^E} vg(v)dv}{q + (1-q)((1-p) + pG(v^E)}.$$

Letting $\gamma(v^E) = \frac{q}{q+(1-q)((1-p)+pG(v^E))}$, we write this expression as:

$$\gamma(v^E)v^E + (1-\gamma(v^E))P^{DYE}(v^E)$$

Recall that $P^{DYE}(v^E)$ increases at a rate less than 1. Thus, the derivative of the price function when the signal is precise is:

$$\overbrace{\gamma(v^E)\cdot 1 + (1-\gamma(v^E))\frac{\partial}{\partial v^E}P^{DYE}(v^E)}^{(i)} + \overbrace{\gamma'(v^E)(v^E - P^{DYE}(v^E))}^{(ii)}.$$

Note that $\gamma'(v^E) < 0$; furthermore, for all $v^E \geq v^D$, we have that $v^E - P^{DYE}(v^E) > 0$ (since $P^{DYE}(v^D) = v^D$ and $P^{DYE}(v^E)$ increases in v^E at a rate less than 1). Thus, (i) is a convex combination of terms less than or equal to 1, and (ii) is negative. Therefore, the sum is less than 1.

Therefore, the expected nondisclosure payoff is a convex combination (with fixed ratios, since the signal is precise with probability q and imprecise with probability $1-q$, independently of v^E) of two terms which increases in v^E at a rate less than 1; thus, the payoff from nondisclosure increases at a rate less than 1 in v^E (for $v^E > v^D$), meaning that there can only be one value in this range where equality holds.

Proof of Proposition 1: Under the conditions of the proposition,

$$\lim_{s\uparrow v^E} P(s,\varnothing) = \frac{qs + (1-q)((1-p)\int_{v_{min}}^{v_{max}} vh(v|s,\underline{\phi})dv + p\int_{v_{min}}^{v^E} vh(v|s,\underline{\phi})dv}{q + (1-q)((1-p) + p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv)},$$

and

$$\lim_{s\downarrow v^E} P(s,\varnothing) = \frac{q(1-p)s + (1-q)((1-p)\int_{v_{min}}^{v_{max}} vh(v|s,\underline{\phi})dv + p\int_{v_{min}}^{v^E} vh(v|s,\underline{\phi})dv)}{q(1-p) + (1-q)((1-p) + p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv)},$$

At $s = v^E$, we have $s > \frac{(1-p)\int_{v_{min}}^{v_{max}} vh(v|s,\underline{\phi})dv + p\int_{v_{min}}^{v^E} vh(v|s,\underline{\phi})dv}{(1-p)+p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv}$, as argued in the proof of Lemma 2. On the other hand, the price function is a convex combination of the right-hand side and the left-hand side of this inequality. Furthermore, inspection of the price function reveals that there is strictly more mass on s for $s < v^E$ than for $s > v^E$. Therefore, the price drops discontinuously at $s = v^E$; thus, as long as the price at s', s'' are sufficiently close to the limits above, we have $P(s', \varnothing) > P(s'', \varnothing)$. On the other hand, as argued in Step One of the proof of Lemma 2, the price function is increasing in s. Therefore, we have $P(s' - \delta, \varnothing) < P(s', \varnothing)$ and $P(s'', \varnothing) < P(s'' + \delta, \varnothing)$, as desired.

Proof of Proposition 2: We first consider the derivative of the price function when $\Delta_\phi = 1$; we then show that the same conclusion of the proposition holds away from but sufficiently close to the limit. Given that all managers with $v > v^E$ disclose, we obtain the following expression for the limiting price:

$$\frac{qs(1-p\mathbb{1}_{s>v^E}) + (1-q)((1-p)\mu + p\int_{v_{min}}^{v^E} vg(v)dv)}{q(1-p\mathbb{1}_{s>v^E}) + (1-q)(1-p+pG(v^E))}. \tag{10}$$

This expression is differentiable whenever $s \neq v^E$, and its derivative is:

$$\frac{q(1-p\mathbb{1}_{s>v^E})}{q(1-p\mathbb{1}_{s>v^E}) + (1-q)(1-p+pG(v^E))}.$$

We have that this expression is smaller for $s > v^E$ than $s < v^E$. Indeed,

$$\begin{aligned} & \frac{q(1-p)}{q(1-p) + (1-q)(1-p+pG(v^E))} < \frac{q}{q + (1-q)(1-p+pG(v^E))} \\ \Leftrightarrow \quad & q^2(1-p) + q(1-p)(1-q)(1-p+pG(v^E)) < q^2(1-p) + (1-q)q(1-p+pG(v^E)) \\ \Leftrightarrow \quad & q(1-p)(1-q)(1-p+pG(v^E)) < (1-q)q(1-p+pG(v^E)) \\ \Leftrightarrow \quad & 1-p < 1, \end{aligned}$$

which holds since $p > 0$ by assumption.

Recall from the previous proofs that when $\Delta_\phi < 1$, the derivative of the price function is

$$\frac{q(1-p\mathbb{1}_{s>v^E}))s+(1-q(1-p\mathbb{1}_{s>v^E}))\left((1-p)\int_{v_{min}}^{v^{max}} vh(v|s,\underline{\phi})dv+p\int_{v_{min}}^{v^E} vh(v|s,\underline{\phi})dv\right)}{q(1-p\mathbb{1}_{s>v^E})+(1-q(1-p\mathbb{1}_{s>v^E}))\left((1-p)+p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv\right)}.$$

For $s \neq v^E$, this is differentiable in s (since h is); differentiating under the integral, we have that the derivative is, for an appropriate constant k,

$$\frac{q(1-p\mathbb{1}_{s>v^E}))+(1-q(1-p\mathbb{1}_{s>v^E}))\left((1-p)\int_{v_{min}}^{v^{max}} v\frac{\partial}{\partial s}h(v|s,\underline{\phi})dv+p\int_{v_{min}}^{v^E} v\frac{\partial}{\partial s}h(v|s,\underline{\phi})dv\right)}{q(1-p\mathbb{1}_{s>v^E})+(1-q(1-p\mathbb{1}_{s>v^E}))\left((1-p)+p\int_{v_{min}}^{v^E} h(v|s,\underline{\phi})dv\right)}$$
$$-k\int_{v_{min}}^{v^E}\frac{\partial}{\partial s}h(v\mid s,\underline{\phi})dv.$$

Note that $\frac{\partial}{\partial s}h(v \mid s, \underline{\phi})$ approaches 0 as $\Delta_\phi \to 1$, by Assumption 1. Since v belongs to a compact interval as well, we have that all terms involving integrals over $\frac{\partial}{\partial s}h(v \mid s, \underline{\phi})$ approach 0 in the limit. In addition, $\int_{v_{min}}^{v_E} h(v \mid s, \underline{\phi})dv$ converges to $G(v^E)$, since the distribution over s converges in distribution to an independent draw from G as $\underline{\phi} \to 0$.

Putting this together, we conclude that $\frac{\partial}{\partial s}P(s, \varnothing)$ converges to (10) as $\Delta_\phi \to 1$. Letting ε denote the difference in the derivative of the price function whenever $\Delta_\phi = 1$, we can find some $\widetilde{\Delta}$ such that whenever $\Delta_\phi > \widetilde{\Delta}$, the derivative of the price function at Δ_ϕ is within $\varepsilon/2$ of its limiting value. For any such Δ_ϕ, the stated inequality will hold, as claimed.

Proof of Lemma 3: When disclosure may occur after the external signal arrives, the market belief will be a function of the signal in the absence of disclosure. We first note that the posterior distribution of v given signal s in the cases covered by the proposition is

$$\begin{cases} (1-q)G(v) & v < s \\ G(v) & v \geq s \end{cases}.$$

In particular, given that this setting coincides with a static disclosure model with the modified distribution over v, existence and uniqueness of threshold equilibrium is standard and follows from arguments in Jung and Kwon (1988); Acharya, DeMarzo, and Kremer (2011); Kartik, Lee, and Suen (2021). This analogy, and the corresponding existing results, imply that there exists a unique function $v^L(s)$ such that the manager remains silent after signal s if and only if $v \leq v^L(s)$.

Proof of Proposition 3: Since the payoff from nondisclosure given s is constant in v, and the payoff from disclosure is increasing in v, the function $v^L(s)$ from Lemma 3 is defined by the indifference condition:

$$v^L(s) = \frac{qs(1-p\mathbb{1}_{s>v^L(s)}) + (1-q)((1-p)\mu + p\int_{v_{min}}^{v^L(s)} vg(v)dv)}{q(1-p\mathbb{1}_{s>v^L(s)}) + (1-q)(1-p+pG(v^L(s)))}.$$

At $s = v^D$, this equation is uniquely satisfied by setting $v^L(s) = v^D$ since $v^D = \frac{(1-p)\mu + p\int_{v_{min}}^{v^D} vg(v)dv}{1-p+pG(v^D)}$. Furthermore, when $s = v^D$, the market's conjecture is independent of the probability that informed managers disclose.

For the next part of the proof we consider an auxiliary game which is identical to ours,[37] with the only difference being that the manager's value is disclosed with exogenous probability $\theta \in [0,1]$ if the signal is precise. Denote the corresponding disclosure threshold by $\widehat{v}_\theta(s)$, which is the implicit solution to the following equation:

$$\widehat{v}_\theta(s)(q(1-p\theta)+(1-q)(1-p+pG(\widehat{v}_\theta(s)))-qs(1-p\theta)-(1-q)((1-p)\mu+p\int_{v_{min}}^{\widehat{v}_\theta(s)} vg(v)dv) = 0. \tag{11}$$

Differentiating the left-hand side with respect to $\widehat{v}_\theta(s)$ yields:

$$\begin{aligned} q(1-p\theta) + (1-q)(1-p+pG(\widehat{v}_\theta(s)) + \widehat{v}_\theta(s)(1-q)pg(\widehat{v}_\theta(s)) - (1-q)p\widehat{v}_\theta(s)g(\widehat{v}_\theta(s))) \\ = q(1-p\theta) + (1-q)(1-p+pG(\widehat{v}_\theta(s)). \end{aligned}$$

On the other hand, the derivative of (11) with respect to s is $-q(1-p\theta)$. Implicit differentiation[38] therefore yields:

$$\frac{d}{ds}\widehat{v}_\theta(s) = \frac{q(1-p\theta)}{q(1-p\theta) + (1-q)(1-p+pG(\widehat{v}_\theta(s))} \in (0,1).$$

With this observation, we return to the original game (where the disclosure decision following an precise signal is endogenous). We are now in position to prove the claims in the proposition:

- We start with (i), the comparative static in q. We note that the price is itself a convex

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[37] We use the auxiliary game as a way of facilitating differentiation of the price function with respect to the signal. Since $\mathbb{1}_{s>v^L(s)}$ is not differentiable, some underlying structure is needed to utilize arguments based on differentiation. Our strategy is to derive this structure in the auxiliary game, where the price function is differentiable. Then, we use this structure to help determine the outcome in the original game.

[38] That is, if a function $f(x)$ is defined by the implicit condition $\Gamma(f(x), x) = 0$, then differentiating with respect to x implies $\Gamma_1 f'(x) + \Gamma_2 = 0$ by chain rule; so $f'(x) = -\frac{\Gamma_2}{\Gamma_1}$, yielding the claimed expression.

combination between s and the expectation of v given $v < v^L(s)$; by log-concavity, the latter increases at a rate less than 1 in $v^L(s)$, which itself increases at a rate less than 1 in s. Thus, whenever $s > v^D$, s is greater than the expectation of v given $v < v^L(s)$. On the other hand, as q increases, relatively more weight is put on s, which implies that if q increases, the threshold must increase as well.

- Next we show (ii). Suppose that $s > v^D$. Then for all θ in the auxiliary game, $\widehat{v}_\theta(s)$ increases in s at a rate less than 1. Since $\widehat{v}_\theta(v^D) = v^D$, we therefore have that $s > \widehat{v}_\theta(s)$. This implies that no matter what probability the market assigns to the informed manager having disclosed, a manager with $v = s$ would prefer to disclose. Therefore, the only possibility for equilibrium in the original game is that the informed manager discloses when the signal is precise (in which case $v = s$). Therefore, $v^L(s) = \widehat{v}_1(s)$. Moreover, since $\widehat{v}_1(s)$ is increasing at a rate less than 1, we conclude that $v^L(s) > v^D$ and $v^L(s) < s$.

  If $s < v^D$, we again use that in the auxiliary game $\widehat{v}_\theta(s)$ is increasing at a rate less than 1 for all s. As a result, $\widehat{v}_\theta(s) > s$ for all θ when $s < v^D$ since $\widehat{v}_\theta(v^D) = v^D$. Therefore, no matter what probability the market assigns to whether informed managers disclose, the manager prefers not to disclose. We therefore have that $v^L(s) = \widehat{v}_0(s)$, and the rest of the argument is analogous to the case where $s > v^D$.

- We now show (iii). Our previous observations regarding the derivative of the price function of the auxiliary game, together with the observation that the outcome in the auxiliary game matches the outcome in the original game, imply that:

$$\frac{d}{ds}v^L(s|s \le v^D) = \frac{q}{q + (1-q)(1-p+pG(v^L(s)))}$$
$$\frac{d}{ds}v^L(s|s > v^D) = \frac{q(1-p)}{q(1-p) + (1-q)(1-p+pG(v^L(s)))}.$$

  Note that as s increases, each of these expressions decreases, since the numerator is constant and the denominator increases. On the other hand, for every s:

$$\frac{q}{q + (1-q)(1-p+pG(v^L(s)))} > \frac{q(1-p)}{q(1-p) + (1-q)(1-p+pG(v^L(s)))}.$$

  Thus, since both expressions are decreasing in s, we have that $\frac{d}{ds}v^L(s|s \le v^D) > \frac{d}{ds}v^L(s|s > v^D)$. That both of these derivatives are in $(0, 1)$ follows from inspection, since the denominator is larger than the numerator and both are positive.

Proof that if $s = v + \varepsilon$ for ε with a log-concave density, then the posterior expectation is increasing in the signal for every prior. The result is standard and follows from Milgrom (1981); the proof is included for completeness. By Milgrom (1981), given a joint distribution $f(s|v)$, the posterior expectation $E[v|s]$ is increasing in s, for any prior over v, if and only if, for $\widetilde{v} > v$, it holds that $\frac{f(s|\widetilde{v})}{f(s|v)}$ is increasing in s. Recall that $s = v + \varepsilon$, for ε distributed according to a log-concave distribution with CDF F. We note that for any s^* we have $\Pr(s \leq s^*|v) = \Pr(v + \varepsilon \leq s^*|v) = \Pr(\varepsilon \leq s^* - v|v) = F(s^* - v)$. So, the density of s given v is $f(s - v)$. Taking $v_1 > v_2$, the derivative of $f(s - v_1)/f(s - v_2)$ with respect to s is positive if and only if:

$$f(s - v_2)f'(s - v_1) - f(s - v_1)f'(s - v_2) > 0,$$

which holds if and only if

$$\frac{f'(s - v_1)}{f(s - v_1)} > \frac{f'(s - v_2)}{f(s - v_2)}.$$

Note that $\frac{d}{ds} \log f(s) = \frac{f'(s)}{f(s)}$, which is decreasing in s by log-concavity. Increasing v from v_2 to v_1 decreases $s - v$, and therefore increases $\frac{f'(s-v)}{f(s-v)}$. Hence, the desired inequality holds.

Proof of Proposition 4: Estimating the effect of external news on the probability of disclosure requires comparing two scenarios: one with external news and one without:

$$\begin{aligned} & \Pr(d = v|\text{no external news}) - \Pr(d = v|\text{external news}) \\ = \; & p \cdot \Pr(d = v|\kappa = I, \text{no external news}) - p \cdot \Pr(d = v|\kappa = I, \text{external news}) \\ \propto \; & \Pr(d = v|\kappa = I, \text{no external news}) - \Pr(d = v|\kappa = I, \text{external news}) \qquad (12) \end{aligned}$$

In the absence of external news, the informed manager discloses whenever $v > v^D$, which occurs with probability $\Pr(d = v|\kappa = I, \text{no external news}) = 1 - G(v^D)$. In the presence of external news, the manager discloses when the observed value exceeds the corresponding threshold $v^L(s)$. But since, upon observing the signal, the manager knows whether the signal is "truth" or "noise," it is instructive to consider two cases:

- If the external signal is "truth," then $v > v^L(s)$ if and only if $v > v^D$ by Proposition 3. Thus, the probability of disclosure conditional on the external signal being precise is $1 - G(v^D)$.
- If the external signal is "noise," then s has density g. Given s, disclosure occurs with probability $1 - G(v^L(s))$. Thus, the probability of disclosure conditional on the signal being imprecise is $\int_{v_{min}}^{v_{max}} (1 - G(v^L(s))g(s)ds$.

Overall, the probability of disclosure in the presence of external news can be expressed as: $\Pr(d = v|\kappa = I, \text{external news}) = q(1 - G(v^D)) + (1 - q)\left(\int_{v_{min}}^{v_{max}}(1 - G(v^L(s)))g(s)ds\right)$. Thus, substituting this term into expression (12) and simplifying the resulting expression, we obtain that

$$\begin{aligned}(12) \quad &= \quad \left(1 - G(v^D)\right) - \left(q(1 - G(v^D)) + (1 - q)\left(\int_{v_{min}}^{v_{max}}(1 - G(v^L(s)))g(s)ds\right)\right) \\ &\propto \quad \int_{v_{min}}^{v_{max}} G(v^L(s))g(s)ds - \int_{v_{min}}^{v_{max}} G(v^D)g(s)ds. \qquad (13)\end{aligned}$$

Taking limits $q \to 1$ and $p \to 0$ jointly, we have that $v^L(s) \to s$ and $v^D \to \mu$; while our model is degenerate in the limit, it is well-defined away from it, and any strict inequalities holding in the limit will hold sufficiently close to it by the dominated convergence theorem (since G is bounded above by 1). Therefore, in the limit, (13) being positive (respectively, negative) becomes (3) (respectively, 4). This observation immediately proves (ii).

For (i), we show that if the inequality holds in the $p \to 0$ limit then crowding out obtains away from the limit. This follows from the observation that $v^D < \mu$ for all $p \in (0, 1)$. Thus, if (3) holds, then since $G(\mu) > G(v^D)$, we also have $\int_{v_{min}}^{v_{max}} G(s)g(s)ds > \int_{v_{min}}^{v_{max}} G(v^D)g(s)ds$. Thus, (13) is positive for all $p \in (0, 1)$ whenever q is sufficiently close to 1, by the same argument as in the previous case.

Proof of Lemma 4: Suppose $c_L < c_E$. Any manager who discloses early obtains $v - c_E$; any manager who does not disclose early obtains at least $v - c_L > v - c_E$ by disclosing late. Thus, any manager who discloses early would have a strictly profitable deviation, meaning this cannot occur in equilibrium.

Proof of Lemma 5: Recall that, given an arbitrary disclosure threshold $\bar{v}$, the market price at a signal s is

$$\frac{qs + (1 - q)((1 - p)\mu + p\int_{v_{min}}^{\bar{v}} vg(v)dv}{q + (1 - q)(1 - p + pG(\bar{v}))}.$$

Note that, for any s satisfying:

$$v^E_{c_L,c_E} - c_L \leq \frac{qs + (1 - q)((1 - p)\mu + p\int_{v_{min}}^{v^E_{c_L,c_E}} vg(v)dv}{q + (1 - q)(1 - p + pG(v^E_{c_L,c_E}))},$$

a manager with $v = v^E_{c_L,c_E}$ will not disclose, since the right-hand side is the nondisclosure payoff. If late disclosure occurs with positive probability, then the manager's payoff at any signal realization inducing late disclosure must be greater than if (a) the market conjectured an early-disclosure threshold of $v^E_{c_L,c_E}$, but (b) late disclosure were not allowed—if not, the manager would prefer to remain silent. Letting $\Pi^{\bar{v}}$ denote the expected nondisclosure payoff of a manager with $v = \bar{v}$ when the market conjectures an early-disclosure threshold of $\bar{v}$ and late disclosure is not allowed, this argument shows the manager's expected payoff is strictly larger than $\Pi^{v^E_{c_L,c_E}}$.

To complete the proof, we consider the following two exhaustive cases:

- If late disclosure never occurs in equilibrium, then $v^E_{c_L,0} = v^E$. If $c_E > 0$, then $v^E_{c_L,c_E}$ will be even larger since the (early) disclosure payoff is decreasing in c_E.

- If late disclosure does occur in equilibrium, then the previous argument implies the manager's nondisclosure payoff is strictly larger than Π^{v^E}. Note that part (ii) of the proof of Lemma 2 shows that, as a function of the (early) disclosure threshold, the expected nondisclosure payoff increases at a rate less than 1. Thus, we have that $\Pi^{\widetilde{v}} > \widetilde{v}$ for any $\widetilde{v} < v^E$; but since the manager's payoff when late disclosure is allowed is weakly greater than $\Pi^{\widetilde{v}}$, we cannot have the manager indifferent between disclosing and remaining silent at any $\widetilde{v} < v^E$. Therefore, to have a threshold equilibrium in this second case, the threshold must increase, implying that $v^E_{c_L,0} > v^E$. The same argument as in the previous bullet shows this implies $v^E_{c_L,c_E} > v^E$ for all $c_E > 0$.

Proof of Proposition 5: For this proof we take $c_E = 0$;[39] Define $\bar{c}$ by:

$$\bar{c} = v^E - \frac{qv_{min} + (1-q)((1-p)\mu + p\int_{v_{min}}^{v^E} vg(v)dv}{q + (1-q)((1-p) + pG(v^E)}.$$

Then, whenever $c_L < \bar{c}$, the manager would disclose late, given early disclosure threshold v^E, for any s realization in some range $[v_{min}, v_{min} + \eta]$. Thus, for any threshold equilibrium given such c_L, disclosure would occur late with positive probability.

Define $\Pi^{\bar{v}}$ as in the proof of Lemma 5, and let $\widetilde{\Pi}^{\bar{v}}$ be the expected payoff of a manager with $v = \bar{v}$ when not disclosing early, assuming the market conjectures all managers with $v > \bar{v}$ disclose early—i.e., $\Pi^{\bar{v}}$ is the threshold manager's payoff when not disclosing early when late disclosure is not allowed, and $\widetilde{\Pi}^{\bar{v}}$ is the payoff if late disclosure *is* allowed. We have that $\Pi^{v^E} = v^E$ by definition, and furthermore that $\Pi^{\widetilde{v}} - \widetilde{v} < 0$ for all $\widetilde{v} > v^E$, with $\Pi^{\widetilde{v}}$ continuous in $\widetilde{v}$. Continuity implies that there exists $\varepsilon_1, \varepsilon_2$ such that $\widetilde{\Pi}^{v^E+\varepsilon_1} < v^E + \varepsilon_1$ whenever $c_L \in (\bar{c} - \varepsilon_2, \bar{c})$, since this inequality holds for all $v > v^E$ when $c_L = \bar{c}$, and since, fixing a conjectured early disclosure threshold, the threshold manager's payoff from not disclosing early is continuous in c_L. But we also have $\widetilde{\Pi}^{v^E} > v^E$ for any such c_L (see

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[39] Note that by continuity the same argument would apply as long as c_E is sufficiently small.

the proof of Lemma 5). Since $\widetilde{\Pi}^{\bar{v}}$ is continuous in $\bar{v}$, it follows from the intermediate value theorem that there exists some c such that equality holds.

We claim that for ε_2 sufficiently small, the threshold $v^E_{c_L,c_E}$ defines an equilibrium early disclosure threshold. Define $s^*(v)$ to be infimum of the set of signals such that a manager with value v discloses late following this signal; if no such signal exists, then we define $s^*(v) = v_{min}$. We consider the following four exhaustive cases:

- $v \in [v_{min}, s^*(v^E_{c_L,0}))$. For c_L sufficiently close to $\bar{c}$, the payoff from not disclosing early is strictly larger than the payoff from disclosing, by continuity (since the nondisclosure price is strictly larger and since $s^*(v^E_{c_L,c_E}) \to v_{min}$ as $c_L \to \bar{c}$). Therefore, ε_2 can be chosen to ensure such manager discloses.

For the other three cases, we argue that the slope of the nondisclosure payoff increases at a rate less than 1. This observation shows that we have an equilibrium, because the manager with $v = v^E_{c_L,0}$ is indifferent between disclosing and not, and since the disclosure payoff increases at a rate equal to 1, all managers with v greater than this threshold prefer disclosure and all managers with v lower than this threshold prefer nondisclosure.

- $v \geq s^*(v^E_{c_L,0})$, but a manager with value v never discloses late in equilibrium. In this case, the non-(early) disclosure payoff is:

  $$qP(v,\varnothing) + (1-q)\left(\int_{v_{min}}^{v_{max}} P(s,\varnothing)g(s)ds\right).$$

  In this case, since a signal of $s = v$ is never disclosed late, we have that the price function is exactly the same as in the baseline model without late disclosure. There, we showed that the price has slope less than 1, which implies that here, the slope of the price function at any such v increases at a rate less than $q < 1$.

- $v \leq v^E_{c_L,0}$ but a manager with value v discloses late with positive probability. Here the non-(early) disclosure payoff is:

  $$qP(v,\varnothing) + (1-q)\overbrace{\left(\int_{v_{min}}^{s^*(v)} (v-c_L)g(s)ds + \int_{s^*(v)}^{v_{max}} P(s,\varnothing)g(s)ds\right)}^{(a)}.$$

  Again, $P(v,\varnothing)$ has a slope less than 1. We differentiate (a) with respect to v, and obtain:

$$(v - c_L - P(s^*(v), \varnothing))(s^*)'(v)g(s^*(v)) + G(s^*(v)).$$

By definition of $s^*(v)$, $v - c_L - P(s^*(v), \varnothing)) = 0$, and as a result this expression simplifies to $G(s^*(v)) < 1$. Thus, the nondisclosure payoff increases at a rate which is a convex combination between two terms less than 1, which is itself less than 1.

- $v > v^E_{c_L,0}$. Since $v^E_{c_L,0} > v^E > v^D$, while the price is discontinuous at $v^E_{c_L,0}$, it has a downward drop, and for $s > v^E_{c_L,0}$ the slope of the price is less than 1. If the manager would not disclose after precise signals, the argument is exactly as in the previous case. If the manager does disclose following precise signals, the payoff is instead:

$$q(v - c_L) + (1 - q)\left(\int_{v_{min}}^{s^*(v)} (v - c_L)g(s)ds + \int_{s^*(v)}^{v_{max}} P(s, \varnothing)g(s)ds\right).$$

  Again, we have that the nondisclosure payoff is a convex combination between a term with slope 1 and a term with slope less than 1, so that the nondisclosure payoff itself has slope no larger than 1.

Therefore, there exists a threshold equilibrium where there is a positive probability that the manager discloses early and a positive probability the manager discloses late. In this equilibrium, $v^E_{c_L,0} > v^E > v^D$ (where the first inequality holds by Lemma 5 and the second by the proof of Lemma 2). But since late disclosure does not occur if $s = v^E_{c_L,0}$, the market's conjecture about the probability the signal is precise following nondisclosure changes as s passes $v^E_{c_L,0}$. Since the threshold being above v^D implies the price jumps downward when the market changes conjectures in the way described (see Proposition 1 and Lemma 2), the price is nonmonotonic at $v^E_{c_L,0}$, with a downward jump, as claimed.

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